PLASTIC ANALYSIS OF REINFORCED CONCRETE TANKS AND SILOS

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By
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to the
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APRIL, 1981

DEDICATED

TO

MY PARENTS

CERTIFICATE

This is to certify that the thesis 'Plastic Analysis of Reinforced Concrete Tanks and Silos', submitted by Mr. Veerendra Kumar, in partial fulfilment of the requirements for the Degree of Master of Technology at the Indian Institute of Technology, Kanpur is a record of bonafide research work carried out under my supervision and guidance. The work embodied in this thesis has not been submitted elsewhere for a degree.

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VEERENDRA KUMAR

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LIST OF SYMBOLS

A_s	-	total cross-sectional area of reinforcement
A_{sc}	-	cross-sectional area of reinforcement in compression
A_{st}	-	cross-sectional area of reinforcement in tension
A_1 to A_5	-	constants of integration
B_1 to B_5	-	constants of integration
CC	-	$4L^2/Rt$, non dimensional geometrical parameter of shells
D	-	diameter
D_s	-	distance between tension and compression steel
K	-	ratio, constant for given steel
L	-	total depth
M_p	-	plastic moment
M_x	-	bending moment
M_{rb}	-	balance moment capacity
$M_{x\theta}$, $M_{\theta x}$	-	twisting moments
N_x	-	axial force
N_p	-	plastic force
N_o	-	limit state force
N_θ	-	circumferential force
$N_{x\theta}$, $N_{\theta x}$	-	membrane shearing force
P_a	-	actual axial capacity of the silo wall along with moment
P_{ao}	-	axial load capacity with zero eccentricity

P_{ab}	-	axial load capacity with eccentricity e_s
R	-	radius
R_1	-	hydraulic radius
T	-	$2\mu' kL/R$, nondimensional friction parameter.
V_x	-	shear force
X	-	depth from top
d	-	effective thickness of shell wall
e	-	X/L
e_b	-	distance of P_{ab} from centre of the shell wall
e_s	-	distance of P_{ab} from tension steel
e_x	-	actual eccentricity
f_{ck}	-	characteristic compressive strength of concrete
f_y	-	characteristic yield strength of steel
k	-	ratio of horizontal static pressure to vertical static pressure due to stored material
$m_x, m_{x1}, m_{x2}, m_{x3}$	-	reduced bending moments
n_x	-	reduced axial stress
n_θ	-	reduced circumferential stress
p	-	horizontal static pressure due to stored material
q	-	vertical static pressure due to stored material
t	-	total thickness of shell wall
u	-	longitudinal displacement
v	-	circumferential displacement
w	-	radial displacement

x,y,z	-	orthogonal cartesian coordinates
z	-	thickness
ϵ	-	axial strain
$\dot{\epsilon}$	-	axial strain rate
ϵ_θ	-	circumferential strain
$\dot{\epsilon}_\theta$	-	circumferential strain rate
ϵ_x	-	longitudinal strain
$\dot{\epsilon}_x$	-	longitudinal strain rate
$\dot{\kappa}_x$	-	curvature rate
α,β	-	coefficient, ratio
σ_y	-	yield stress in tension or compression
σ_r	-	tensile rapture stress of concrete
σ_r'	-	crushing stress of concrete in compression
γ	-	weight per unit volume of stored material
μ'	-	coefficient of friction between stored material and wall
μ_θ	-	ratio of total circumferential steel to concrete
$\mu_{\theta t}$	-	ratio circumferential steel in tention to concrete
$\mu_{\theta c}$	-	ratio circumferential steel in compression to concrete
μ_x	-	ratio of total axial steel to concrete
μ_{xc}	-	ratio of axial steel in compression to concrete
μ_{xt}	-	ratio of axial steel in tension to concrete
ϕ	-	angle of repose of stored material, diameter of of reinforcement bar
$\dot{\lambda},\dot{\phi}$	-	reduced strain rate
η	-	reduced abscissa.

ABSTRACT

In the present literature, the storage structures whether they are water tanks or silos, are designed as per 'elastic' theory with the advent of limit state design methods. There is a great need for the limit design of such structures. In this thesis cylinderical water tanks and storage silos made of reinforced concrete are analysed as per the plastic theory based upon the lower bound theorem. The pressure distribution considered in the case of silos are due to Janssen's and Reimbert's. The collapse modes are described and design method is illustrated with an example.

1. INTRODUCTION

1.1 HISTORY OF CIVIL ENGINEERING

Civil engineering is a branch of human activity, which has been constantly pursued from times immemorial, when people began to adopt the environment for meeting their needs. In the ancient times the structures were built by artisans. When nomadic tribes inhabited around the world, the storage of essentials like water, food etc. did not pose any problem, as they never thought of either storing or hoarding. With the growth of civilization people not only settled at some places but also cultivated land to meet their livelihood. This change in lifestyle brought into focus the need for storage structures. Earthen pots were evolved for purposes of carrying and storing water besides storing grains . As the population increased, the need for larger storage capacity arose. Thus various types of storage structure made of variety of materials had come into vogue. The cylindrical water tanks and large silos fall under this category[(1)].

The 'knowhow' of things is known as science and 'doing' things is called art. The blend of these two is engineering. Civil engineering is the oldest branch of

engineering. Present days civil engineers, structural engineers in particular, are constantly improving the various design procedures, as the behaviour of the materials is very well understood with advances in science.

1.2 IDEALIZATION OF STRUCTURES

An element or an assembly of elements formed into a shape to resist a set of external forces is called a structure(2).

A structure is referred to as a planar or a space structure depending on whether or not the centre line of the elements forming the structure lie in a single plane. All structures are invariably three dimensional, but depending on the relative ratios of sides, they are treated as one, two or three-dimensional. Whenever it is possible, three dimensional structures are reduced to lower dimensional category for purposes of analysis and design. Shell, a space structure but circular cylindrical axisymmetrically loaded, is idealized as a one dimensional structure for analysis and design.

1.3 STRUCTURAL ANALYSIS

Evaluation of the stress resultants, namely, internal resisting forces and deformations of a structure

caused by external forces, is known as structural analysis. The purpose of structural analysis is to study or predict the response or behaviour of a loaded structure. The entire structural analysis is founded on four basic principles, namely the equilibrium of forces, the conditions of compatibility, the force-deformation relationship of the material used and the known boundary conditions. If all the four conditions are satisfied, the solution is exact[2]. In this study, the solution found is exact.

The analysis may be broadly classified as 'elastic' or 'plastic', depending on the range of mechanical behaviour considered. Elastic analysis predicts the behaviour at working loads. The stress at any point in the domain of of the structure must not exceed certain permissible working stresses. Plastic analysis predicts the behaviour at collapse. The stresses at certain points or along the lines reach the yield stress of the material. In this study, plastic analysis is used to analyse the tanks and silos.

1.4 STRUCTURAL DESIGN

The primary purpose of structural design is force transmission through solid matter with certain design objectives such as structural safety, rigidity, serviceability,

or geometric, functional and aesthetic requirements. But from the engineers'viewpoint, there are two main criteria for the adequacy of a structure which must be satisfied in the most economical fashion: Safety, that is sufficient strength to resist applied loads without danger of collapse and serviceability, that is the ability to carry loads without excessive deformation or local distress.

The strength requirement of the structure may be satisfied in several ways. In the 'conventional' or 'elastic' design which is also known as 'working stress design', the strength is checked by ensuring that no point of the structure is stressed above a value called the allowable or working stress which is calculated by using factor of safety on the yield stress of the material. The actual factor of safety varies from structure to structure depending upon the degree of indeterminancy, boundary conditions and the nature of loading.

In the plastic design the total load is investigated under which the structure will collapse. The safety of the structure is controlled by the choice of a load factor, defined as the ratio of the collapse load to the working load. In this,the behaviour of the structure at working load is not taken into account.

In the limit state design, the design load and the design stresses are found using partial safety factor on load acting on the structure and the characteristic strength of materials. The aim of this design is to achieve acceptable probabilities that the structure will not become unfit for the use for which it is intended. Thus this appears to be the most rational method of design to date.

1.5 MECHANICAL BEHAVIOUR OF ENGINEERING STRUCTURES

Structures respond in different ways when subjected to external influences depending on their geometric characteristics and on the mechanical and other properties of the material used. The mechanical properties of a material may be represented by determining the functional relation between the stresses and strains.

For most materials encountered in engineering practice, it is practically impossible to devise a single set of mathematical equations that realistically describe the interplay of instantaneous elastic response, after-effect and hysteresis, creep and plastic flow. It is also a well established fact that unique relations do not exist in between stress and strain components in the plastic region. The strain depends not only on the

final state of stress but also on the loading history. So the stress-strain relations which is used in elasticity must be replaced by increments of stress and strain in plasticity.

Steel and concrete are the most common materials used in structural engineering practice. The strain hardening portion of stress-strain curve is neglected in the present study. Hence the theoretical computations are always on the conservative side of the experimental values when the idealization is elastic perfectly plastic or rigid plastic.

1.6 LITERATURE SURVEY

Airy[(3)] derived the equations for pressure distribution in shallow bins for granular material based on the Coulomb's theory of earth pressure and modified this for deep bins. Janssen[(3)] also derived equations for pressure distribution in deep bins which has been followed by Reimbert's deep bin theory[(4)]. Basically now a days the static pressure is calculated by Janssen's classical theory or Reimbert's modern deep bin theory. All these pressure distribution theory gives static pressure.

Prante[(5)], in 1896, made the first measurement on full scale silos. Hoffmann[(5)], in 1916, pointed out that these

results showed a lower wall pressure during filling than the pressure computed from Janssen's theory, but this pressure rises during emptying.

Subsequent investigations by Bovey[5] and Lufft[5] in 1904, led to the conclusion that Janssen's formula as supplemented by Konen in 1896 could, provided that the component coefficients were suitably chosen, be considered to indicate the wall pressure when filling and only moderate increase were noted during emptying.

There are two types of flow condition, dynamic and nondynamic. The first flow condition gives a large pressure increase than the latter flow condition. The flow depends upon the filling condition.

The Reimbert's equations give a much better correlation with experimental data for static and filling condition than the traditional Janssen's deep bin theory. The emptying condition varies widely with the type of material and must be considered separately[6].

Save[7] and Sawczuk[8] have done some work on the plastic analysis of tanks and silos for constant hoop reinforcement only. From the foregoing review it is clear that not much work has been done on the plastic analysis of tanks /silos with variable hoop reinforcement. In this

study tanks and silos are analysed for varying circumferential reinforcement. In the case of silos the pressure distribution considered are due to Janssen and Reimbert.

1.7 SCOPE OF THE THESIS

The thesis is divided into seven chapters. Chapter 1 introduces the fundamental concepts of structural analysis and structural design.

In chapter 2 the analysis, equilibrium equations and yield condition of a axisymmetrically loaded circular cylinder shell are presented.

Chapter 3 deals with the analysis of circular cylindrical tanks, while chapter 4 and chapter 5 deal with the analysis of silos for Reimbert's and Janssen's pressure distributions respectively.

In chapter 6 all the results of chapter 5 have been simplified for ultimate load design method and the design of a silo is presented.

In chapter 7 the results obtained in the preceding chapters have been critically evaluated and conclusions drawn.

2. ESSENTIALS OF AXISYMMETRICALLY LOADED CYLINDRICAL SHELLS

2.1 DEFINITION

A shell is a curved plate. The surface which bisects the thickness of plate is called the middle surface. If the middle surface and the thickness are specified at each point then a shell is completely defined geometrically. The thickness of a shell is considered to be very small compared to the other dimensions of the shell and the radii of curvature. Hence a shell can be generalised as a two dimensional structure[9].

The surface of a shell is obtained by rotating a curve of given shape around a fixed axis. For cylindrical shell this curve is a straight line.

2.2 EQUILIBRIUM EQUATIONS

To establish the equilibrium equation an element as shown in Fig. 2.2.1 is considered. The shell of radius R is subjected to a radial pressure p and an axial pressure q. Due to symmetry the membrane shearing forces $N_{x\theta} = N_{\theta x}$ vanish in this case and the forces N_θ are constant along the circumference. From symmetry it can be concluded that only the forces V_x do not vanish. Also from symmetry it

can be concluded that the twisting moments $M_{x\theta} = M_{\theta x}$ vanish and the bending moment M_θ are constant along the circumference. Under such conditions of symmetry three (two of rotation and one of translation) of the six (three of rotation and three of translation) equations of equilibrium of the element are identically satisfied, and as such only the remaining three equations should be considered. These are obtained by projecting the forces on x and z axes and taking the moment about the y axis. External forces consist of pressure normal to the surface and vertically downward force. These three equations of equilibrium are[9]

$$\frac{dV_x}{dx} + \frac{N_\theta}{R} = p \tag{2.2.1}$$

$$V_x = \frac{dM_x}{dx} \tag{2.2.2}$$

$$\frac{dN_x}{dx} = q \tag{2.2.3}$$

Substituting equation (2.2.2) in equation (2.2.1) to eliminate V_x from equation (2.2.1)

$$\frac{d^2M_x}{dx^2} + \frac{N_\theta}{R} = p \tag{2.2.4}$$

Let M_p and N_p be the plastic moment capacity per unit length and axial force capacity per unit length respectively.

Thus $$M_p = \sigma_y t^2/4 ,$$

$$N_p = \sigma_y t,$$

where σ_y is yield stress of material and t is the thickness.

Introducing the non-dimensional parameters

$$p^* = \frac{pR}{\sigma_y t} ,$$

$$q^* = \frac{q}{\sigma_y t} ,$$

$$m_x = \frac{M_x}{M_p} ,$$

$$n_\theta = \frac{N_\theta}{N_p} ,$$

$$n_x = \frac{N_x}{N_p} ,$$

the equations (2.2.3) and (2.2.4) reduce to

$$\frac{dn_x}{dx} = q^* \qquad (2.2.5)$$

$$\frac{tR}{4} \frac{d^2 m_x}{dx^2} + n_\theta = p^* \qquad (2.2.6)$$

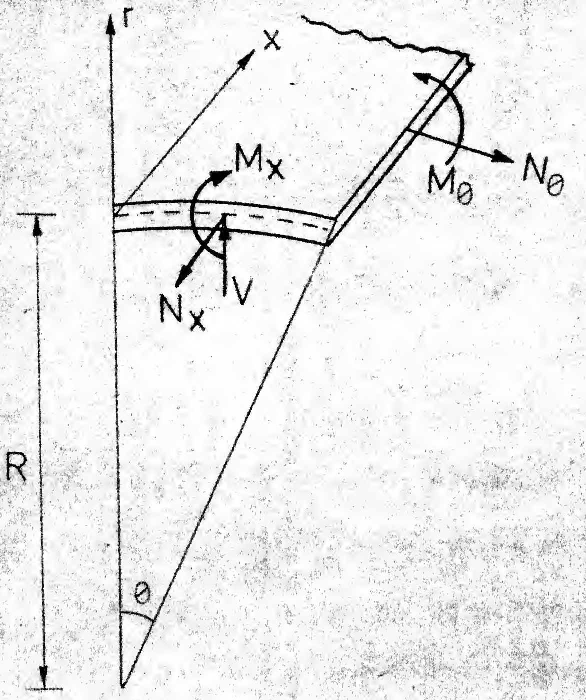


Fig. 2·3·1 Typical shell element

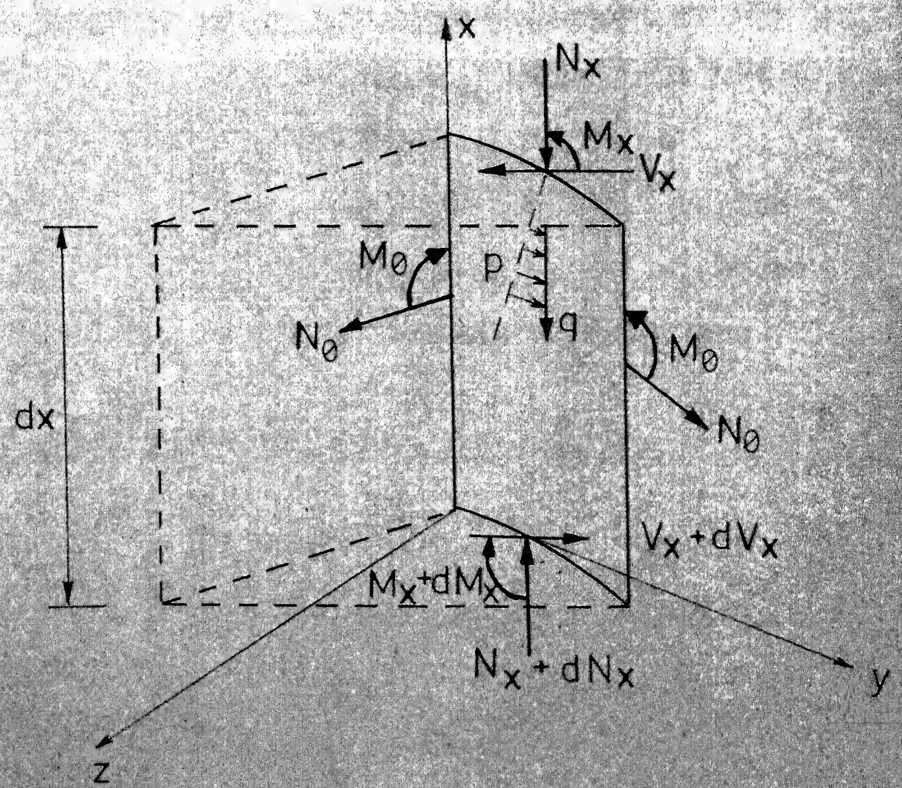


Fig. 2·2·1 Typical shell element

2.3 SPECIFIC POWER OF DISSIPATION

Consider a cylindrical coordinate system x, θ and r as shown in Fig. 2.3.1 and also a cylindrical shell of length L and radius R. Because of the symmetry of revolution, circumferential displacements, v, vanish, whereas longitudinal displacements, u, and radial displacements, w (positive inward) are functions of x only. Also due to rotational symmetry, the circumferential curvature rate, $\dot{\kappa}_\theta$, vanishes. There is no circumferential displacement. Any point of the shell is displaced in the meridian plane in which it is contained. Hence, any two neighbouring meridian planes experience no relative rotation, and M_θ does no work. So $\kappa_\theta = \dot{\kappa}_\theta = 0$. The radius of the median surface varies from R to R + w, where w is the radial displacement of the median surface, the circumferential strain,

$$e_\theta = \frac{w}{R-z} \quad (-t/2 \leq z \leq t/2)$$

must be regarded as constant (and hence $\kappa_\theta = 0$) because z (thickness) is negligible with respect to R from the very definition of a shell.

Rates of transversal shear vanishes because it is assumed that the material normals remain normal to the

deformed median surface. So the dissipation rate is

$$D \;=\; M_x \quad {}_x + N_x \, \dot{e}_x + N_\theta \, \dot{e}_\theta \qquad (2.3.1)$$

where M_x, N_x, and N_θ are generalized stresses, given by the bending moment, axial force and circumferential force respectively per unit length.

The corresponding generalized strain rates are

$$\begin{aligned} {}_x \;&\doteq\; \text{curvature rate} = -\frac{d^2\dot{w}}{dx^2} \quad , \\ \dot{e}_x \;&=\; \text{longitudinal strain rate} \;=\; \frac{d\dot{u}}{dx} \quad , \\ \dot{e}_\theta \;&=\; \text{circumferential strain rate} = -\frac{\dot{w}}{R} \quad , \qquad (2.3.2) \end{aligned}$$

So the equation (2.3.1) will be

$$D = -\, M_x \frac{d^2\dot{w}}{dx^2} + N_x \frac{d\dot{u}}{dx} - N_\theta \frac{\dot{w}}{R} \qquad (2.3.3)$$

using the'reduced stresses'

$$\begin{aligned} n_x \;&=\; N_x/N_p \quad , \\ m_x \;&=\; M_x/M_p \quad , \\ n_\theta \;&=\; N_\theta/N_p \quad , \qquad (2.3.4) \end{aligned}$$

where the axial force capacity of the shell element per unit length, $N_p = \sigma_y t$ and the plastic moment capacity of the shell element per unit length, $M_p = \sigma_y t^2/4$, where t=thickness

of the shell, and σ_y is yield stress of the shell material.

Therefore, the relation (2.3.3) reduces to

$$D = \sigma_y(m_x(-\frac{t^2}{4}\ \frac{d^2\dot{w}}{dx^2}) + n_x t\ \frac{d\dot{u}}{dx} + n_\theta\ (-\frac{t\dot{w}}{R}\ )) \qquad (2.3.5)$$

Hence, the generalised strain rates corresponding to the reduced stresses are

$$\dot{\phi}_x = -\ \frac{t^2}{4}\ \frac{d^2\dot{w}}{dx^2} \quad ,$$

$$\dot{\lambda}_x = t\ \frac{d\dot{u}}{dx} \quad ,$$

$$\dot{\lambda}_\theta = -t\ \frac{\dot{w}}{R} \quad ,$$

the relation (2.3.5) is written as (7)

$$D = \sigma_y\ (m_x\ \dot{\phi}_x + n_x\ \dot{\lambda}_x + n_\theta\ \dot{\lambda}_\theta) \qquad (2.3.6)$$

2.4 YIELD CONDITION

It is assumed that the concrete is perfectly plastic with the yield curve in plane stress as shown in Fig. 2.4.1, where σ_r and σ'_r are the tensile and compressive yield stresses respectively. Here the compressive forces and stresses will be regarded as positive, as is usual in reinforced concrete practice. Strain rates will be

positive when corresponding to contraction. Also longitudinal and circumferential directions are principal directions.

It is assumed that the amount of reinforcement subjected to compression is small enough to contribute in a negligible manner to the compressive strength, and that the reinforcement subjected to tension has a yield stress σ_y and cross-sectional area A_x and A_θ per unit of length. Because of symmetry of revolution, the strain rates are distributed across the shell thickness t as shown in Fig. 2.4.2(a) and 2.4.2(b) for longitudinal and circumferential direction respectively. For a stationary process, the strain rate distribution is alike. The strain in any layer is $\epsilon_i = \lambda_i - z\,\kappa_i$, so it is expressed in terms of elongation λ_i of the middle surface of the shell, its curvature κ_i and the distance z from the middle surface. Due to radial symmetry no circumferential curvature κ_θ occurs.

Applying the normality law to the strain rates of Fig. 2.4.2, it can be concluded that, for the concrete, the stress point must be in C (Fig. 2.4.1) for $-\eta t/2 \leq z \leq t/2$ because $\dot{\epsilon}_1 < 0$ and $\dot{\epsilon}_2 < 0$ and in D (Fig. 2.4.1) for $-t/2 \leq z \leq -\eta t/2$ because $\dot{\epsilon}_1 > 0$ and $\dot{\epsilon}_2 < 0$. During failure the reinforcement yields in both

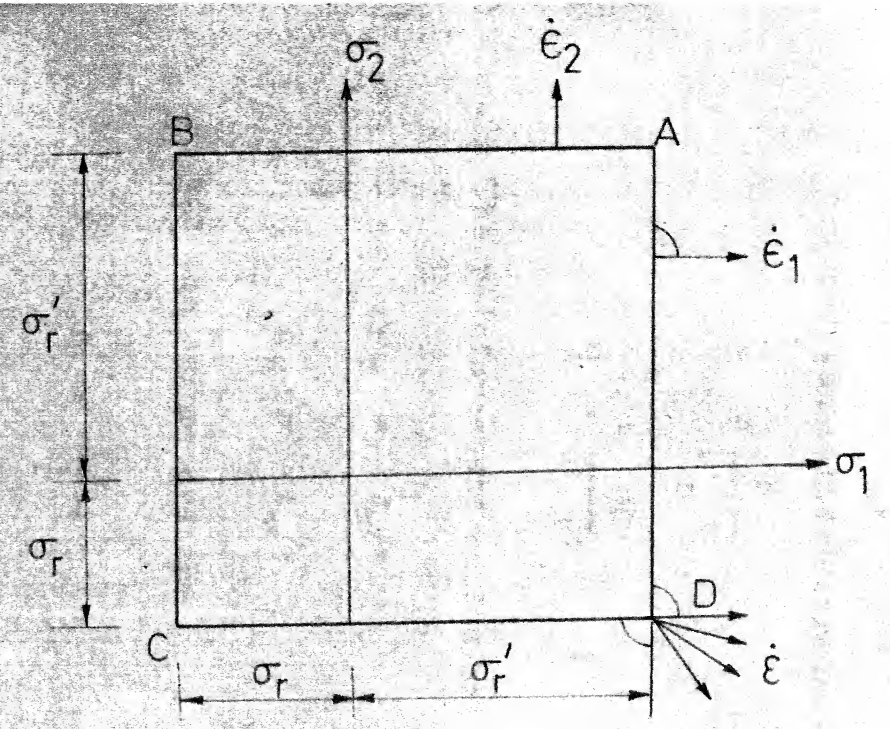


Fig. 2·4·1 Square yield criterion

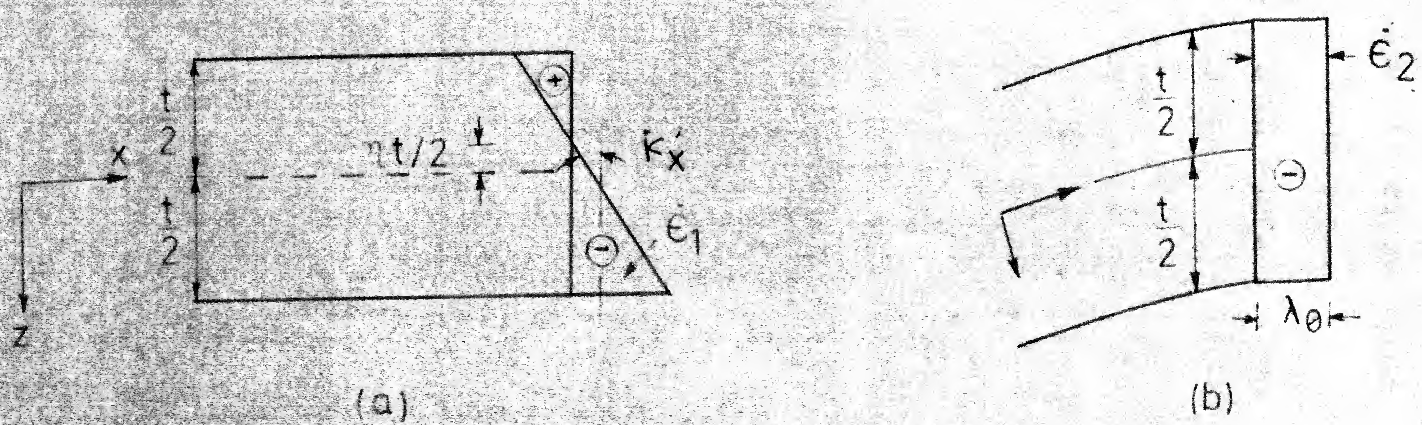


Fig. 2·4·2 Curvature and strain across a cross-section of silo wall

axial and circumferential directions. The generalized stresses M_x, N_x, and N_θ can be evaluated for every value of the parameter η. Elimination of η from the expression of M_x, N_x and N_θ gives the yield surface. The reduced generalized stresses are[10]

$$n = \frac{N}{\sigma'_r \, t}$$

$$m = \frac{4M}{\sigma'_r \, t^2} \; ;$$

The reinforcement ratios are

$$\mu_x = A_x/t$$

$$\mu_\theta = A_\theta/t \; ;$$

and the parameters are

$$\alpha = \frac{\sigma_r}{\sigma'_r}$$

$$\beta = \frac{\sigma_y}{\sigma'_r}$$

where t is thickness of the shell, σ'_r is compressive yield stress of concrete, σ_r is tensile yield stress of concrete, and σ_y yield stress of reinforcement.

The yield surface is formed of two parabolic cylinders given by :

$$m_x(1+\alpha) + 2n_x^2 + 2n_x\,(2\beta\mu_x + \alpha-1) + 2\beta^2\mu_x^2 - 4\beta\mu_x - 2\alpha = 0\,,$$

$$\text{for } m_x > 0 \qquad (2.4.1)$$

$$-m_x(1+\alpha) + 2n_x^2 + 2n_x\,(\alpha-1) - 2\alpha = 0\,, \text{ for } m_x < 0 \qquad (2.4.2)$$

This cylinder is bounded by the planes with equations

$$n_\theta = 1 \quad \text{for compression} \qquad (2.4.3)$$

and

$$n_\theta = -\alpha-\beta\mu_\theta \quad \text{for tension} \qquad (2.4.4)$$

The tensile strength of concrete is neglected so $\alpha = 0$. In the case of tank $N_x = 0$ but in the case of silos N_x is not equal to zero. It is considered in design only. Hence the analysis has been restricted to m_x and n_θ plane. The interaction of the surface with the n_θ and m_x for given n_x will represent the actual condition of failure. The corresponding boundary of the safe domain in the form of a rectangle ABCD is shown in Fig. 2.4.4 for $n_x = 0$.

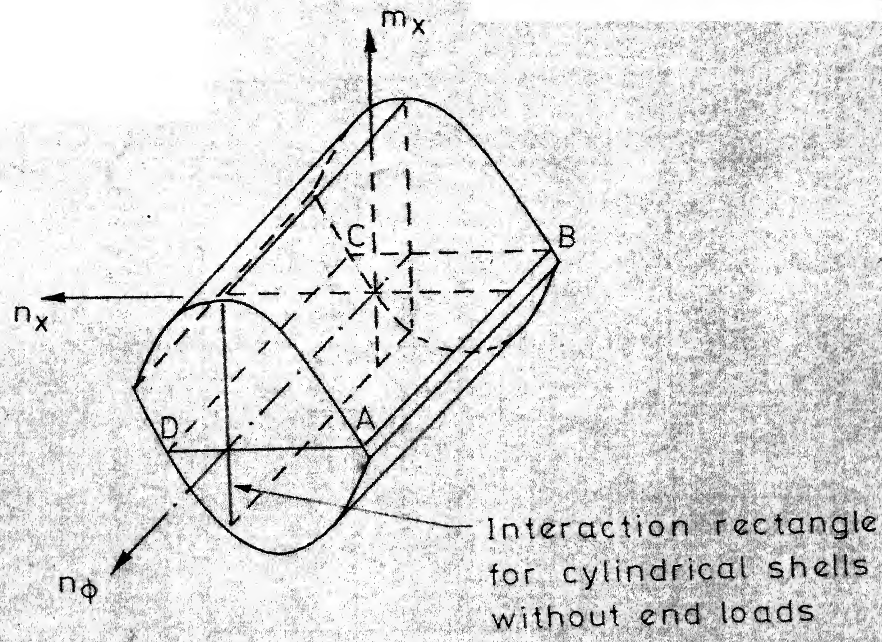


Fig. 2·4·3 Yield surface for reinforced concrete cylindrical shells

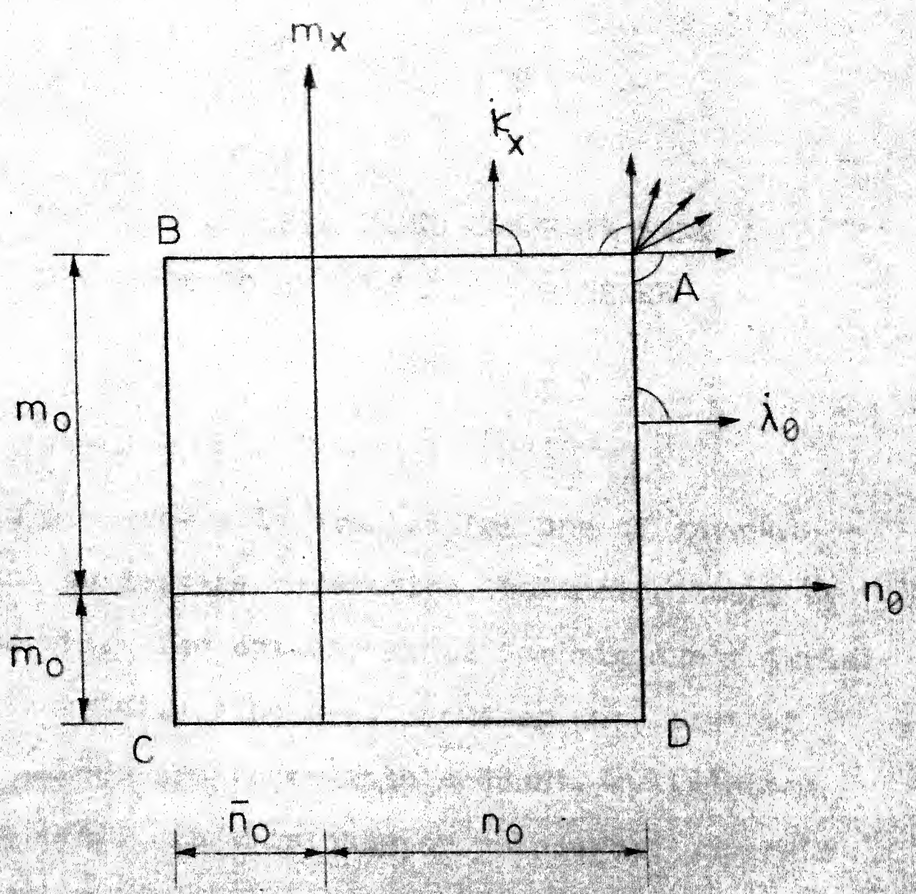


Fig. 2·4·4 Square yield criterion

3. ANALYSIS OF REINFORCED CONCRETE TANK SUBJECTED TO HYDROSTATIC PRESSURE

3.1 GENERAL

Tanks are generally erected for one of several purposes, such as service reservoirs, balancing tanks or storage tanks etc. The optimal shape for minimum material consumption is usually a sphere. However when cost of erection and others are taken into account, cylindrical tanks are the best. In this chapter reinforced concrete cylindrical tanks subjected to hydrostatic pressure are analysed. The most important aspect besides strength is the leak proofness. Thus when a fluid storage tank is designed, the width of cracks on fluid face must be controlled. The porosity of the walls has to be minimised by using a rich mix of concrete[3].

3.2 EQUILIBRIUM EQUATIONS

In the case of tank, which is subjected to hydrostatic pressure p, the vertical force q is zero. So the dimensionless equations (2.3.5) and (2.3.6) will be[7]

$$\frac{dn_x}{dx} = 0 \qquad (3.2.1)$$

$$\frac{tR}{4} \frac{d^2 m_x}{dx^2} + n_\theta = p^* \qquad (3.2.2.)$$

where

$$p^* = pR/\sigma'_r\, t$$

$$p = \gamma X$$

$$p_o = \gamma L$$

$$p^* = (p_o R/\sigma'_r\, t)\, e = p_o^*\, e$$

where

$$e = X/L$$

$$p_o^* = p_o R/\sigma'_r\, t$$

γ = unit weight of stored liquid

R = radius of meridian surface

t = thickness of the shell wall.

Now equation (3.2.2) will be

$$\frac{tR}{4L^2} \frac{d^2 m_x}{de^2} + n_\theta = p_o^* e \qquad (3.2.3)$$

Let the non dimensional geometrical parameter of shell be

$$CC = \frac{4L^2}{tR}$$

Then equation (3.2.3) transforms to

$$\frac{d^2 m_x}{de^2} + CC \cdot n_\theta = CC \; p_o^* \; e \qquad (3.2.4)$$

3.3 CLASSIFICATION OF TANKS

The tanks are classified into three catogeries namely shallow, medium, and deep. This classification is based upon the mode of failure of the tank at collapse. The typical modes of failure are shown in Fig. 3.3.1. The mode of failure depends upon the value of the non-dimensional parameter of the shell CC.

3.4 METHOD OF ANALYSIS

Following are the steps for analysing the tank:

(i) The distribution of circumferential stress resultant n_θ is assumed.

(ii) The equation of equilibrium (3.2.4) is solved for a given value of the shell parameter CC.

(iii) The constants of integration are determined by using various boundary and the continuity conditions.

Thus the distribution of bending moment is determined. Care should be taken to see that the shear and moment are continuous at the critical sections.

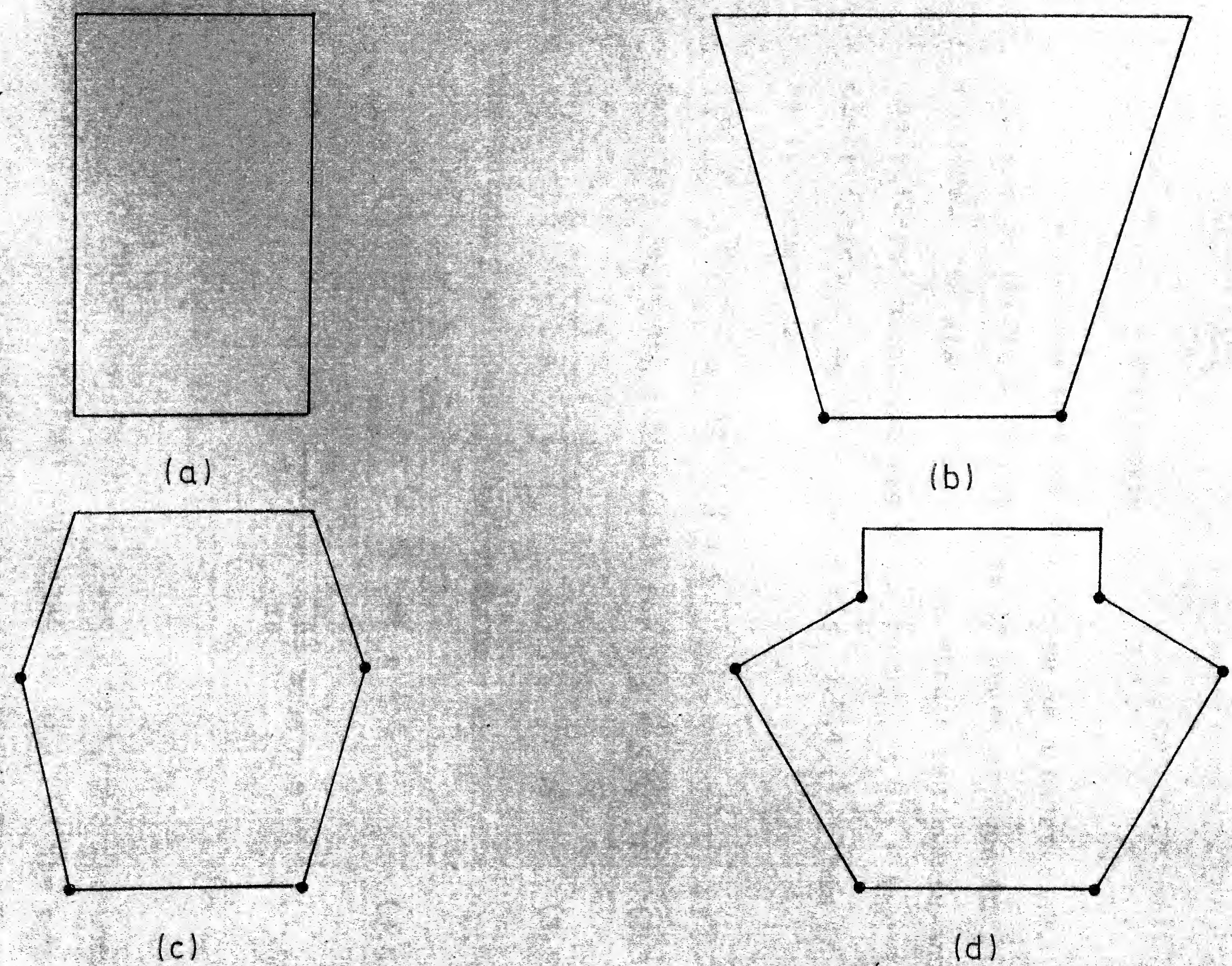


Fig.3·3·1 Typical modes of failure of tanks

3.5 SHALLOW TANK

3.5.1 Stress Distribution

A shallow tank is also known as a short tank. The failure of the tank at collapse is known as mode one failure. The distribution of the stress resultants and the collapse mechanism are shown in Fig. 3.5.1. The distribution of n_θ is shown in Fig. 3.5.1(c). This is given by the equations

Zone I : $n_\theta = \alpha n_o$ for $0 \leq e \leq \beta$

Zone II: $n_\theta = \alpha n_o + K n_o\ (e-\beta)$ for $\beta \leq e \leq 1$ (3.5.1)

where $$K = \frac{1-\alpha}{1-\beta} \qquad (3.5.2)$$

The boundary and continuity conditions are

$$m_{x1} = 0\ , \quad \frac{dm_{x1}}{de} = 0 \quad \text{at} \quad e = 0 \qquad (3.5.3)$$

and $$m_{x1} = m_{x2} \text{ and } \frac{dm_{x1}}{de} = \frac{dm_{x2}}{de} \text{ at } e = \beta \qquad (3.5.4)$$

3.5.2 $0 \leq e \leq \beta$ (Zone I)

In this zone, which is from 0 to β, the circumferential reinforcement is constant and equal to αn_o. So the equilibrium equation (3.2.4) reduces to

$$\frac{d^2 m_{x1}}{de^2} = CC\ (p_o^* e - \alpha n_o) \qquad (3.5.5)$$

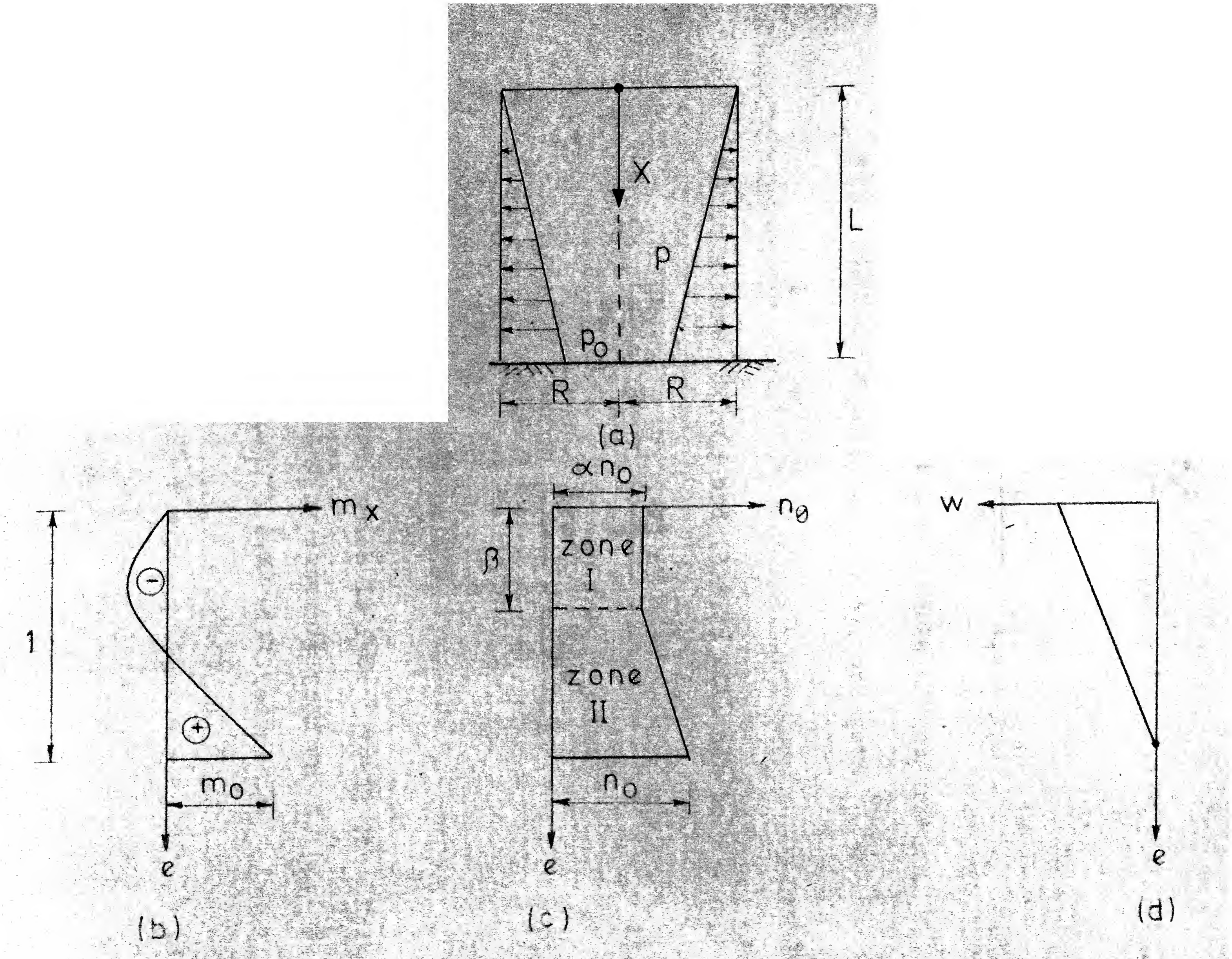


Fig. 3.5.1 Stress distribution and collapse mechanism for short tank

Integrating once

$$\frac{dm_{x1}}{de} = CC\ (\frac{p_o^* e^2}{2} - \alpha n_o e) + A_1 \qquad (3.5.6)$$

Integrating once again

$$m_{x1} = CC\ (\frac{p_o^*\ e^3}{6} - \frac{\alpha n_o e^2}{2}) + A_1 e + B_1 \qquad (3.5.7)$$

where A_1 and B_1 are the constants of integration.

From boundary condition (3.5.3), one gets

$$A_1 = B_1 = 0 \qquad (3.5.8)$$

So equations (3.5.6) and (3.5.7) will now be

$$\frac{dm_{x1}}{de} = CC\ (\frac{p_o^*\ e^2}{2} - \alpha n_o e) \qquad (3.5.9)$$

$$m_{x1} = CC\ (\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2}) \qquad (3.5.10)$$

3.5.3 $\beta \leq e \leq 1$ (Zone II)

In this zone, which is from β to 1, the circumferential reinforcement is varying linearly from αn_o to n_o. So the equilibrium equation (3.2.4) will reduce to

$$\frac{d^2 m_{x2}}{de^2} = CC\ (\ p_o^*\ e - \alpha n_o - K n_o\ (\ e - \beta\ )) \qquad (3.5.11)$$

First integration gives

$$\frac{dm_{x2}}{de} = CC\left(\frac{p_o^* e^2}{2} - \alpha n_o e - Kn_o\left(\frac{e^2}{2} - \beta e\right)\right) + A_2 \qquad (3.5.12)$$

Second integration gives

$$m_{x2} = CC\left(\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2} - Kn_o\left(\frac{e^3}{6} - \frac{\beta e^2}{2}\right)\right) + A_2 e + B_2 \qquad (3.5.13)$$

where A_2 and B_2 are constants of integration.

From the continuity conditions (3.5.4)

$$A_2 = -CC\frac{Kn_o\beta^2}{2} \qquad (3.5.14(a))$$

$$B_2 = CC\frac{Kn_o\beta^3}{6} \qquad (3.5.14(b))$$

So equations (3.5.12) and (3.5.13) will be

$$\frac{dm_{x2}}{de} = CC\left(\frac{p_o^* e^2}{2} - \alpha n_o e - \frac{Kn_o}{2}(e-\beta)^2\right) \qquad (3.5.15)$$

$$m_{x2} = CC\left(\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2} - \frac{Kn_o}{6}(e-\beta)^3\right) \qquad (3.5.16)$$

There is one more boundary condition

$$m_{x1} = m_o \quad \text{at} \quad e = 1 \qquad (3.5.17)$$

Using this in equation (3.5.16)

$$m_o = \frac{CC}{6}\left(p_o^* - 3\alpha n_o - Kn_o(1-\beta)^3\right) \qquad (3.5.18)$$

and after simplification one gets

$$p_o^* = \frac{6m_o}{CC} + 3\alpha n_o + Kn_o\ (1-\beta)^3 \qquad (3.5.19)$$

By using the equations (3.5.10) and (3.5.16), the bending moment distribution will be as shown in Fig. 3.5.1(b).

3.5.4 Critical Condition

As the L/R ratio of the shell increases, the value of CC will also increase and from equation (3.5.19) it is clear that the value of p_o^* will decrease for increase in the value of CC. From equation (3.5.16) one can infer that the increase in CC will cause decrease in m_x and for some value of e, and CC, m_x will reach the maximum negative value[10]. So for certain value of $e = e_o$

$$m_x = -\bar{m}_o \text{ and } \frac{dm_x}{de} = 0 \qquad (3.5.20)$$

when $e_o \leq \beta$

$$m_{x1} = -\bar{m}_o \text{ and } \frac{dm_{x1}}{de} = 0 \text{ at } e = e_o$$

This gives

$$e_o = \frac{2\alpha n_o}{p_o^*} \qquad (3.5.21)$$

$$\text{and} \quad CCR = \frac{6\bar{m}_o}{3\alpha n_o e_o^2 - p_o^* e_o^3} \qquad (3.5.22)$$

where CCR is the critical value of CC.

When $e_o \geq \beta$

$$m_{x2} = -\bar{m}_o \text{ and } \frac{dm_{x2}}{de} = 0 \text{ at } e = e_o$$

This gives

$$e_o = \frac{(\alpha n_o - Kn_o\beta) \pm ((\alpha n_o - Kn_o\beta)^2 + Kn_o(p_o^* - Kn_o)\beta^2)^{1/2}}{(p_o^* - Kn_o)} \qquad (3.5.23)$$

$$CCR = \frac{6\bar{m}_o}{(e_o^3(Kn_o - p_o^*) + 3n_o e_o^2(\alpha - K\beta) + 3Kn_o\beta^2 e_o - Kn_o\beta^3)} \qquad (3.5.24)$$

3.5.5 Special Case

Let $m_o = \bar{m}_o = n_o = 1$

Then $p_o^* = 6/CC + 3\alpha$

For zone 1, $0 \leq e \leq \beta$

$$e_o = 2\alpha/p_o^*$$

$$CCR = 6/(3\alpha e_o^2 - p_o^* e_o^3) \qquad (3.5.25)$$

For zone 2, $\beta \leq e \leq 1$

$$e_o = \frac{(\alpha - K\beta) \pm ((\alpha - K\beta)^2 + K(p_o^* - K)\beta^2)^{1/2}}{(p_o^* - K)}$$

$$CCR = \frac{6}{(e_o^3(K-p_o^*) + 3e_o^2\ (\alpha - K\beta) + 3K\beta^2 e_o - K\beta^3)} \qquad (3.5.26)$$

Let $\alpha = \beta = 1$

Then $K = 0$

$$p_o^* = 3\ (\ 1 + 2/CC)$$

$$e_o = 2/p_o^*$$

$$CCR = \frac{6}{3e_o^2 - p_o^* e_o^3} \qquad (3.5.27)$$

3.5.6 Results

The distribution of n_θ is dependent upon the values of α and β. For a given set of α and β values the critical shell parameter value CCR, the location of hinge e_o, and the collapse pressure p_o^* are shown in Table 3.1 for $m_o = \bar{m}_o = n_o = 1$.

TABLE 3.1 : CCR FOR FIRST MODE OF FAILURE OF TANK AT $m_o = \bar{m}_o = n_o = 1$

α	β	CCR	e_o	p_o^*
0.25	0.25	118.5	0.52999	1.22251
0.50	0.25	39.6	0.58203	1.93500
0.50	0.50	38.0	0.56509	1.78289
0.75	0.50	23.3	0.58631	2.57037
1.00	1.00	17.1	0.59683	3.35088

3.6 MEDIUM TANK

3.6.1 Stress Distribution

A medium tank is such that there is hoop compression at the free end. This type of failure is designated as mode two failure and this is termed as 'total failure' or total collapse. The distribution of stress resultants and collapse mechanism are shown in Fig. 3.6.1. The distribution of n_θ is as shown in Fig. 3.6.1(b). There are three zones:

$$\begin{aligned}
&\text{Zone I}: && n_\theta = -\alpha\bar{n}_o && \text{for } 0 \le e \le e_1 \\
&\text{Zone II}: && n_\theta = \alpha n_o && \text{for } e_1 \le e \le \beta \\
&\text{Zone III}: && n_\theta = \alpha n_o + K(e-\beta)\, n_o && \text{for } \beta \le e \le 1
\end{aligned} \tag{3.6.1}$$

The boundary and the continuity conditions are

$$m_{x1} = 0, \quad \frac{dm_{x1}}{de} = 0 \quad \text{at } e = 0 \tag{3.6.2}$$

$$m_{x1} = m_{x2}, \quad \frac{dm_{x1}}{de} = \frac{dm_{x2}}{de} \quad \text{at } e = e_1 \tag{3.6.3}$$

$$m_{x2} = m_{x3}, \quad \frac{dm_{x2}}{de} = \frac{dm_{x3}}{de} \quad \text{at } e = \beta \tag{3.6.4}$$

The continuity conditions at $e = e_2$ depend whether or not

$$e_2 \gtrless \beta$$

If $\quad e_2 \le \beta$

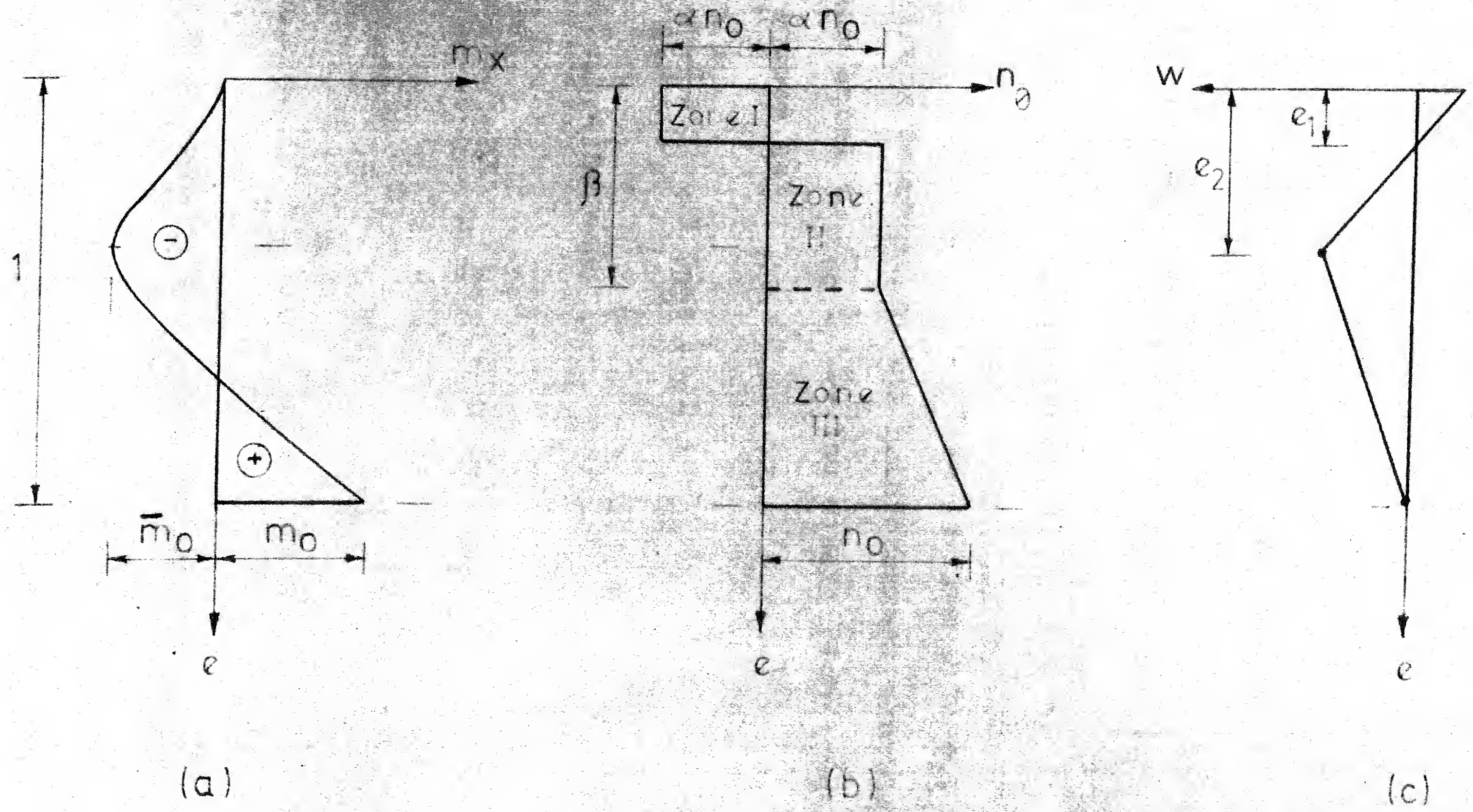


Fig. 3·6·1 Stress distribution and collapse mechanism for medium tank

$$m_{x2} = -\bar{m}_o \;, \quad \frac{dm_{x2}}{de} = 0 \quad \text{at} \quad e = e_2$$

and if

$$e_2 \geq \beta$$

$$m_{x3} = -\bar{m}_o \;, \quad \frac{dm_{x3}}{de} = 0 \quad \text{at} \quad e = e_2 \tag{3.6.5}$$

$$m_{x3} = m_o \quad \text{at} \quad e = 1 \tag{3.6.6}$$

3.6.2 $0 \leq e \leq e_1$ (Zone I)

In this zone, which is from 0 to e_1, the circumferential stress is negative and reinforcement provided is constant and equal to $\alpha\bar{n}_o$. So the equilibrium equation (3.2.4) reduces to

$$\frac{d^2m_{x1}}{de^2} = CC\ (p_o^*e + \alpha\bar{n}_o) \tag{3.6.7}$$

Integrating once

$$\frac{dm_{x1}}{de} = CC\ \left(\frac{p_o^*e^2}{2} + \alpha\bar{n}_o e\right) + A_1 \tag{3.6.8}$$

Integrating once again

$$m_{x1} = CC\ \left(\frac{p_o^*e^3}{6} + \frac{\alpha\bar{n}_o e^2}{2}\right) + A_1 e + B_1 \tag{3.6.9}$$

where A_1 and B_1 are constants of integration.

From boundary conditions (3.6.2)

$$A_1 = B_1 = 0 \tag{3.6.10}$$

So equations (3.6.8) and (3.6.9) will be

$$\frac{dm_{x1}}{de} = CC\left(\frac{p_o^* e^2}{2} + \alpha \bar{n}_o e\right) \tag{3.6.11}$$

and $$m_{x1} = CC\left(\frac{p_o^* e^3}{6} + \frac{\alpha \bar{n}_o e^2}{2}\right) \tag{3.6.12}$$

3.6.3 $e_1 \leq e \leq \beta$ (Zone II)

In this zone, which is from e_1 to β, the circumferential stress is positive and reinforcement provided is constant equal to αn_o from e_1 to β. So the equilibrium equation (3.2.4) will be

$$\frac{d^2 m_{x2}}{de^2} = C^2 \left(p_o^* e - \alpha n_o\right) \tag{3.6.13}$$

From first integration, one gets

$$\frac{dm_{x2}}{de} = CC\left(\frac{p_o^* e^2}{2} - \alpha n_o e\right) + A_2 \tag{3.6.14}$$

From second integration, one gets

$$mx_2 = CC\left(\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2}\right) + A_2 e + B_2 \tag{3.6.15}$$

where A_2 and B_2 are constants of integration.

Using the continuity conditions (3.6.3)

$$A_2 = CC\,\alpha e_1 (n_o + \bar{n}_o)$$

$$B_2 = -\frac{CC\,\alpha e_1^2}{2}(n_o + \bar{n}_o) \tag{3.6.16}$$

So equations (3.6.14) and (3.6.15) reduce to

$$\frac{dm_{x2}}{de} = CC\left(\frac{p_o^* e^2}{2} - \alpha n_o e + \alpha e_1 (n_o + \bar{n}_o)\right) \qquad (3.6.17)$$

and

$$m_{x2} = CC\left(\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2} + \alpha e_1 e (n_o + \bar{n}_o) - \frac{\alpha e_1^2}{2}(n_o + \bar{n}_o)\right) \qquad (3.6.18)$$

3.6.4 $\beta \leq e \leq 1$ (Zone III)

In this zone, which is from β to 1, the circumferential stress is positive and reinforcement provided is linearly varying from αn_o to n_o from β to 1 . So the equilibrium equation (3.2.4) reduces to

$$\frac{d^2 m_{x3}}{de^2} = CC\left(p_o^* e - \alpha n_o - Kn_o (e-\beta)\right) \qquad (3.6.19)$$

First integration gives

$$\frac{dm_{x3}}{de} = CC\left(\frac{p_o^* e^2}{2} - \alpha n_o e - Kn_o\left(\frac{e^2}{2} - \beta e\right)\right) + A_3 \qquad (3.6.20)$$

Second integration gives

$$m_{x3} = CC\left(\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2} - Kn_o\left(\frac{e^3}{6} - \frac{\beta e^2}{2}\right)\right) + A_3 e + B_3 \qquad (3.6.21)$$

where A_3 and B_3 are constants of integration.

From the continuity conditions (3.6.4)

$$A_3 = CC\left(\alpha e_1 (n_o + \bar{n}_o) - \frac{Kn_o \beta^2}{2}\right)$$

$$B_3 = CC\left(\frac{Kn_o\beta^3}{6} - \frac{\alpha e_1^2}{2}(n_o + \bar{n}_o)\right) \qquad (3.6.22)$$

So equations(3.6.20) and (3.6.21) reduce to

$$\frac{dm_{x3}}{de} = \left(\frac{p_o^* e^2}{2} - \alpha n_o e - Kn_o\left(\frac{e^2}{2} - \beta e\right) + \alpha e_1(n_o+\bar{n}_o) - \frac{Kn_o\beta^2}{2}\right. \qquad (3.6.23)$$

$$m_{x3} = CC\left(\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2} - Kn_o\left(\frac{e^3}{6} - \frac{\beta e^2}{2}\right) + \alpha e_1 e\,(n_o + \bar{n}_o)\right.$$
$$\left. - \frac{Kn_o e\beta^2}{2} + \frac{Kn_o\beta^3}{6} - \frac{\alpha e_1^2}{2}(n_o + \bar{n}_o)\right) \qquad (3.6.24)$$

Using boundary condition (3.6.6) in equation (3.6.24) one gets

$$p_o^* = \frac{6m_o}{CC} + 3\alpha n_o + Kn_o\,(1-\beta)^3 - (\bar{n}_o + n_o)\,(6\alpha e_1 - 3\alpha e_1^2) \qquad (3.6.25)$$

From the first condition of (3.6.5)

$$p_o^* = \frac{2(\alpha n_o e_2 - \alpha e_1(n_o + \bar{n}_o))}{(e_2^2)} \qquad (3.6.26)$$

and

$$p_o^* = \frac{(-6\bar{m}_o/CC + 3\alpha n_o e_2^2 - 6\alpha e_1 e_2\,(n_o + \bar{n}_o) + 3\alpha e_1^2(n_o + \bar{n}_o))}{(e_2^3)} \qquad (3.6.27)$$

From the second condition of (3.6.5)

$$p_o^* = \frac{(2\alpha n_o e_2 - 2\alpha e_1(n_o + \bar{n}_o) + Kn_o(e_2-\beta)^2)}{(e_2^2)} \qquad (3.6.28)$$

and

$$p_o^* = \frac{(-6\bar{m}_o/CC + 3\alpha n_o e_2^2 - 6\alpha e_1 e_2(n_o+\bar{n}_o)+2\alpha e_1^2(n_o+\bar{n}_o)+Kn_o(e_2-\beta)^3)}{e_2^3}$$

(3.6.29)

By using equations (3.6.12), (3.6.15) and (3.6.24), the bending moment distribution will be as shown in Fig. 3.6.1(a).

3.6.5 Critical Condition

As the value of CC increases the maximum value of positive moment increases at certain value of CC at $e = e_3$. This value of CC is critical. This is clear from the equation (3.6.12). The value of CCR will also depend on whether $e_3 \gtrless \beta$. The value of e_1 is confined upto 0.1L of the upper portion, so there is no point in considering $\beta < e_1$.

So at $e = e_3$

$$m_x = +m_o \quad \text{and} \quad \frac{dm_x}{de} = 0 \qquad (3.6.30)$$

when

$$e_3 \leq \beta$$

$$e_3 = \frac{2\alpha n_o}{p_o^*} - e_2 \qquad (3.6.31)$$

$$CCR = \frac{6m_o}{(p_o^* e_3^3 - 3\alpha n_o e_3^2 + 6\alpha e_1 e_3(n_o+\bar{n}_o) - 3\alpha e_1^2(n_o+\bar{n}_o))}$$

(3.6.32)

and when $e_3 \geq \beta$

$$e_3 = \frac{2n_o(\alpha - K\beta)}{(p_o^* - Kn_o)} - e_2 \qquad (3.6.33)$$

$$CCR = \frac{6m_o}{(p_o^* e_3^3 - 3\alpha n_o e_3^2 + 6\alpha e_1 e_3 (n_o + \bar{n}_o) - 3\alpha e_1^2 (n_o + \bar{n}_o) - Kn_o (e_3 - B)^3)} \qquad (3.6.34)$$

3.6.6 Special Case

When $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

$$p_o^* = 6/CC + 3\alpha + K(1-\beta)^3 - 6\alpha e_1(2-e_1) \qquad (3.6.35)$$

when $e_2 \leq \beta$ then equations (3.6.26)and (3.6.27) will be

$$p_o^* = \frac{-6/CC + 3\alpha e_2^2 - 12\alpha e_1 e_2 + 6\alpha e_1^2}{e_2^3} \qquad (3.6.36)$$

$$e_2 = \frac{\alpha \pm (\alpha^2 - 4\alpha p_o^* e_1)^{1/2}}{p_o^*} \qquad (3.6.37)$$

When $e_2 \geq \beta$ then equations (3.6.28) and (3.6.29) reduce to

$$p_o^* = \frac{-6/CC + 3\alpha e_2^2 + K(e_2-\beta)^3 - 12\alpha e_1 e_2 + 6\alpha e_1^2}{e_2^3} \qquad (3.6.38)$$

$$e_2 = \frac{(\alpha-K\beta) \pm ((\alpha-K\beta)^2 - (p_o^*-K)(4\alpha e_1 - K\beta^2))^{1/2}}{(p_o^* - K)} \qquad (3.6.39)$$

When $e_3 \leq \beta$ equations (3.6.31) and (3.6.32) will be

$$e_3 = \frac{2\alpha}{p_o^*} - e_2 \tag{3.6.40}$$

$$CCR = \frac{6}{(p_o^* e_3^3 - 3\alpha e_3^2 + 12\alpha e_1 e_3 - 6\alpha e_1^2)} \tag{3.6.41}$$

When $e_3 \geq \beta$ then equations (3.6.33) and (3.6.34) will be

$$e_3 = \frac{2(\alpha - K\beta)}{p_o^* - K} - e_2 \tag{3.6.42}$$

$$CCR = \frac{6}{(p_o^* e_3^3 - 3\alpha e_3^2 + 12\alpha e_1 e_3 - 6\alpha e_1^2 - K(e_3 - \beta)^3)} \tag{3.6.43}$$

When $\alpha = \beta = 1$

$$p_o^* = 6/CC + 6e_1^2 - 12e_1 + 3 \tag{3.6.44}$$

$$e_2 = \frac{2 \pm \sqrt{1 - 4p_o^* e_1}}{p_o^*} \tag{3.6.45}$$

$$e_3 = \frac{2}{p_o^*} - e_2 \tag{3.6.46}$$

$$CCR = \frac{6}{(p_o^* e_3^3 - 3e_3^2 + 12e_1 e_3 - 6e_1^2)} \tag{3.6.47}$$

3.6.7 Critical Values

The distribution of n_θ is dependent upon the values of α and β. For a given set of α and β values the critical

shell parameter value CCR, location of hinges e_2 and e_3, the range of hoop compression from 0 to e_1, and the collapse pressure p_o^* are shown in Table 3.2 for $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$.

TABLE 3.2 : CCR FOR SECOND MODE OF FAILURE OF TANK FOR $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

α	β	CCR	e_1	e_2	e_3	p_o^*
0.25	0.25	-	-	-	-	-
0.50	0.25	250.15	0.075	0.736	0.2071	1.37369
0.50	0.50	-	-	-	-	-
0.75	0.50	147.20	0.0862	0.7377	0.16436	1.610233
1.00	1.00	93.10	0.0890	0.7446	0.2349	2.043000

As the value of CC approaches CCR the medium shell becomes deep shell and failure mode changes from second mode to third mode.

3.7 DEEP TANK

3.7.1 Stress Distribution

This tank is also known as long tank. The failure of the tank at collapse is known as mode three failure. The

distribution of the stress resultants and the collapse mechanism are shown in Fig. 3.7.1. This mode of collapse is only partial. The upper portion of the tank remains intact. The distribution of n_θ is shown in Fig. 3.7.1(b). This is given by the equations

$$\text{Zone I} : n_\theta = \alpha n_o \quad \text{for} \quad e_4 \leq e \leq \beta$$

$$\text{Zone II} : n_\theta = \alpha n_o + K(e-\beta)\, n_o \quad \text{for} \ \beta \leq e \leq 1 \qquad (3.7.1)$$

The boundary and the continuity conditions are

$$m_{x1} = m_o , \quad \frac{dm_{x1}}{de} = 0 \quad \text{at} \quad e = e_4 \quad \text{for } e_4 \leq \beta$$

$$m_{x2} = m_o , \quad \frac{dmx_2}{de} = 0 \quad \text{at} \quad e = e_4 \text{ for } e_4 \geq \beta \qquad (3.7.2)$$

$$m_{x1} = m_{x2} , \quad \frac{dm_{x1}}{de} = \frac{dm_{x2}}{de} \quad \text{at } e = \beta \text{ for } e_4 \leq \beta \qquad (3.7.3)$$

The continuity conditions at $e = e_5$ depends whether

$$e_5 \gtrless \beta$$

For $e_5 \leq \beta$

$$m_{x1} = -\bar{m}_o , \quad \frac{dm_{x1}}{de} = 0 \text{ at } e = e_5$$

and for

$$e_5 \geq \beta$$

$$m_{x2} = -\bar{m}_o , \quad \frac{dm_{x2}}{de} = 0 \quad \text{at} \quad e = e_5 \qquad (3.7.4)$$

and

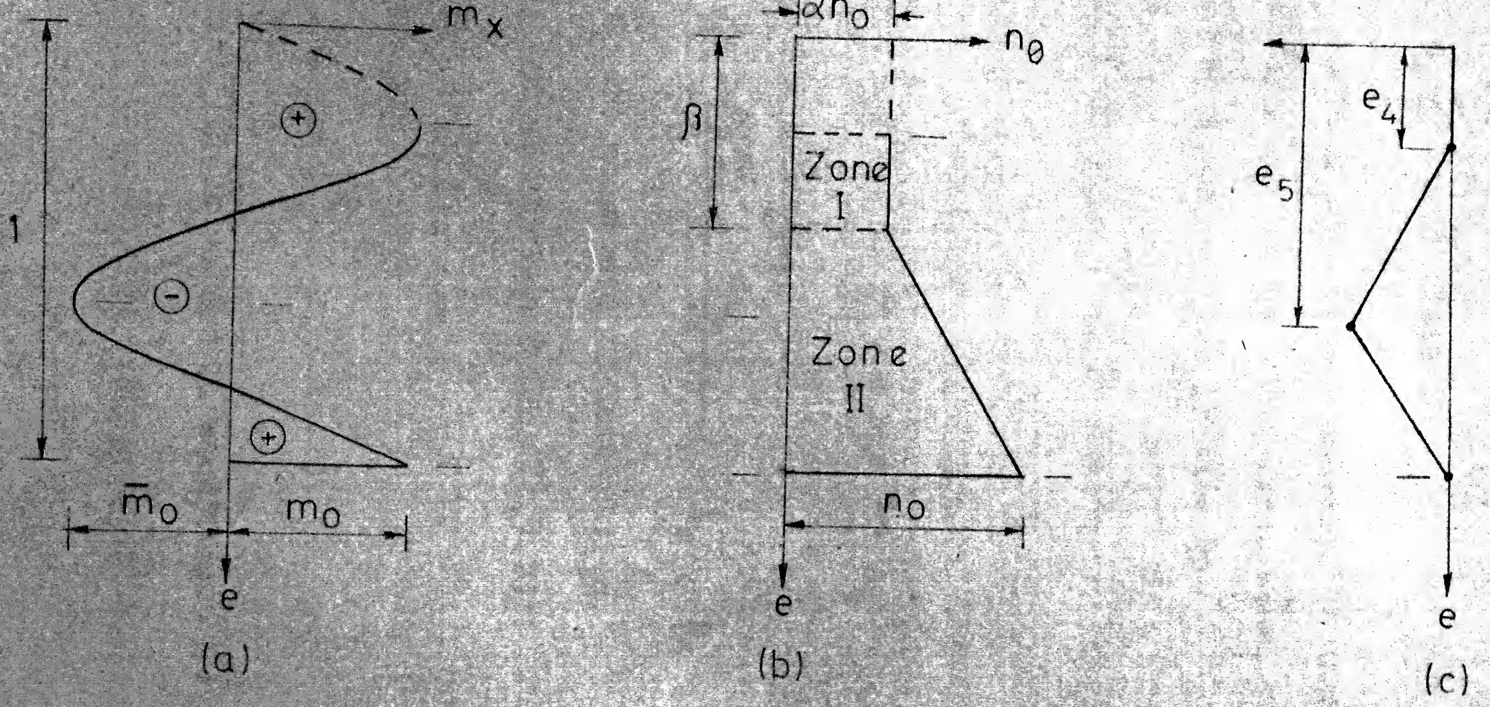


Fig. 3·7·1 Stress distribution and collapse mechanism for deep tank

$$m_{x2} = m_o \quad \text{at} \quad e = 1 \tag{3.7.5}$$

3.7.2 $e_4 \leq e \leq \beta$ (Zone I)

In this zone, circumferential stress is positive and reinforcement is constant throughout, from e_4 to β, is αn_o. So equation (3.2.4) will be

$$\frac{d^2 m_{x1}}{de^2} = CC\ (p_o^* e - \alpha n_o) \tag{3.7.6}$$

Integrating once

$$\frac{dm_{x1}}{\alpha e} = CC\ (p_o^* \frac{e^2}{2} - \alpha\ n_o e\ ) + A_4 \tag{3.7.7}$$

Integrating once again

$$mx_1 = CC\ (\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2}) + A_4 e + B_4 \tag{3.7.8}$$

A_4 and B_4 are constants of integration.

From the continuity conditions (3.7.2)

$$A_4 = -\ CC\ (\frac{p_o^* e_4^2}{2} - \alpha n_o e_4)$$

$$B_4 = m_o + CC\ (\frac{p_o^* e_4^3}{3} - \frac{\alpha n_o e^2}{2}) \tag{3.7.9}$$

Substituting these values in equations (3.7.7) and (3.7.8) one gets,

$$\frac{dm_{x1}}{de} = CC\left(\frac{p_o^* e^2}{2} - \alpha n_o e - \frac{p_o^* e_4^2}{2} + \alpha n_o e_4\right) \qquad (3.7.10)$$

$$m_{x1} = CC\left(\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2} - \frac{p_o^* e_4^2 e}{2} + \alpha n_o e_4 e + \frac{p_o^* e_4^3}{3} - \frac{\alpha n_o e_4^2}{2}\right) + m_o$$

(3.7.11)

3.7.3 $\beta \leq e \leq 1$ (Zone II)

In this zone circumferential stress is positive from β to 1 , and the reinforcement provided in this region is linearly varying from αn_o to n_o. So the equation (3.7.4) will reduce to

$$\frac{d^2 m_{x2}}{de^2} = CC\left(p_o^* e - \alpha n_o - K n_o (e-\beta)\right) \qquad (3.7.12)$$

First integration give

$$\frac{dm_{x2}}{de} = CC\left(\frac{p_o^* e^2}{2} - \alpha n_o e - K n_o\left(\frac{e^2}{2} - \beta e\right)\right) + A_5$$

(3.7.13)

Second integration gives

$$m_{x2} = CC\left(\frac{p_o^* e^3}{6} - \frac{\alpha n_o e^2}{2} - K n_o\left(\frac{e^3}{6} - \frac{\beta e^2}{2}\right)\right) + A_5 e + B_5$$

(3.7.14)

where A_5 and B_5 are constants of integration.

From the continuity condition (3.7.3)

$$A_5 = CC\ (\alpha n_o e_4 - \frac{p_o^* e_4^2}{2} - \frac{Kn_o \beta^2}{2}) \tag{3.7.15}$$

$$B_5 = \quad + CC\ (\frac{p_o^*\ e_4^3}{3} - \frac{\alpha n_o e_4}{2} + \frac{Kn_o \beta^3}{6}) \tag{3.7.16}$$

Substituting these values in equations (3.7.13) and (3.7.14) one gets

$$\frac{dm_{x2}}{de} = CC\ (\frac{p^*}{2}(e^2 - e_4^2) - \alpha n_o (e - e_4) - \frac{Kn_o}{2}\ (e-\beta)^2) \tag{3.7.17}$$

and

$$m_{x2} = CC\ (\frac{p_o^*}{2}\ (\frac{e^3}{3} - e_4^2 e + \frac{e_4^3}{3}) - \frac{\alpha n_o}{2}\ (e-e_4)^2 - \frac{Kn_o}{6}(e-\beta)^3) + m_o \tag{3.7.18}$$

Using boundary condition (3.7.5).

$$p_o^* = \frac{3\alpha n_o (1-e_4)^2 + Kn_o (1-\beta)^3}{(1+2e_4)\ \ (1-e_4)^2} \tag{3.7.19}$$

3.7.4 $e_4 \leq \beta$ (Case I)

For $e_5 \leq \beta$, using the continuity conditions (3.7.4) in equation (3.7.11), one gets

$$p_o^* = 2\alpha n_o / (e_4 + e_5) \tag{3.7.20}$$

and

$$CC = \frac{6(m_o + \bar{m}_o)}{(\alpha n_o (3e_5^2 - 6e_4 e_5 + 3e_4^2) - p_o^*\ (e_5^3 - 3e_4^2 e_5 + 2e_4^3\ ))} \tag{3.7.21}$$

when $\beta \leq e_5 \leq 1$, using the continuity conditions (3.7.4) in equation (3.7.18), one gets

$$p_o^* = \frac{2\alpha n_o(e_5-e_4) + Kn_o(e_5-\beta)^2}{(e_5^2 - e_4^2)} \qquad (3.7.22)$$

$$CC = \frac{6(m_o+\bar{m}_o)}{(3\alpha n_o(e_5-e_4)^2+Kn_o(e_5-\beta)^3-p_o^*(e_5^3-3e_4^2e_5+2e_4^3))} \qquad (3.7.23)$$

3.7.5 $\beta \leq e_4$ (Case II)

For this condition equilibrium equation (3.2.4) is

$$\frac{d^2m_{x3}}{de^2} = CC\ (p_o^*\ e - \alpha n_o - Kn_o(e-\beta)) \qquad (3.7.24)$$

First integration gives

$$\frac{dm_{x3}}{de} = CC(\frac{p_o^*e^2}{2} - \alpha n_o e - Kn_o(\frac{e^2}{2} - \beta e)) + A_6 \qquad (3.7.25)$$

Second integration gives

$$m_{x3} = CC\ (\frac{p_o^*e^3}{6} - \frac{\alpha n_o e^2}{2} - Kn_o(\frac{e^3}{6} - \frac{\beta e^2}{2})) + A_6 e + B_6 \qquad (3.7.26)$$

where A_6 and B_6 are constants of integration.

Using the continuity conditions (3.7.2), one gets

$$A_6 = -\,CC\,\left(\frac{p_o^* e_4^2}{2} - \alpha n_o e_4 - Kn_o\left(\frac{e_4^2}{2} - \beta e_4\right)\right) \tag{3.7.27}$$

and

$$B_6 = m_o - CC\,\left(-\frac{p^*{}_o e_4^3}{3} + \frac{\alpha n_o e_4^2}{2} + Kn_o\left(\frac{e_4^3}{3} - \frac{\beta e_4^2}{2}\right)\right) \tag{3.7.28}$$

From the boundary conditions (3.7.5)

$$p_o^* = \frac{3\alpha n_o(1-e_4)^2 + Kn_o(1-3\beta+6\beta e_4-3\beta e_4^2-3e_4^2+2e_4^3)}{(1-3e_4^2+2e_4^3)} \tag{3.7.29}$$

and from the continuity condition (3.7.4)

$$e_5 + e_4 = 2(\alpha - K\beta)/(p_o^* - K) \tag{3.7.30}$$

$$CC = \frac{6(m_o+\bar{m}_o)}{(3\alpha n_o(e_5-e_4)^2 + Kn_o(e_5^3-3\beta e_5^2-3e_4^2 e_5+6\beta e_5 e_4+2e_4^3-3\beta e_4^2) - p_o^*(e_5^2-3e_4^2 e_5 + 2e_4^3))} \tag{3.7.31}$$

3.7.6 Special Case

Let $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

For $e_4 \leq \beta$ equation (3.5.19) will be

$$p_o^* = (K(1-\beta)^3 + 3\alpha(1-e_4)^2)/((1+2e_4)(1-e_4)^2) \tag{3.7.32}$$

When $e_5 \leq \beta$, equation (3.7.20) and (3.7.21) will be

$$p_o^* = 2\alpha/(e_5+e_4) \tag{3.7.33}$$

and

$$CC = 12/(3\alpha(e_4-e_5)^2-p_o^*(e_5^3-3e_4^2e_5+2e_4^3)) \qquad (3.7.34)$$

when $\beta \leq e_5$, equations (3.7.22) and (3.7.23) reduce to

$$p_o^* = (2\alpha(e_5-e_4)+K(e_5-\beta)^2)/(e_5^2-e_4^2) \qquad (3.7.35)$$

and

$$CC = 12/(3\alpha(e_5-e_4)^2+K(e_5-e_4)^3-p_o^*(e_5^3-3e_4^2e_5+2e_4^3)) \qquad (3.7.36)$$

when $e_4 \geq \beta$, the equations (3.7.29) , (3.7.30) and (3.7.31) will be

$$p_o^* = \frac{(3\alpha(1-e_4)^2+K(1-3\beta+6\beta e_4-3\beta e_4^2-3e_4^2+2e_4^3\ ))}{(1+2e_4)\ (1-e_4)^2} \qquad (3.7.37)$$

$$CC = \frac{12}{(3\alpha(e_5-e_4)^2+K(e_5^3-3\beta e_5^2-3e_4^2e_5+6\beta e_4e_5 + 2e_4^3 -3\beta e_4^2) - p_o^*(e_5^3-3e_4^2e_5+2e_4^3))} \qquad (3.7.38)$$

and $\quad e_4 + e_5 = (2(\alpha-K\beta))/(p_o^*-K) \qquad (3.7.39)$

when $\alpha = \beta = 1$

$$p_o^* = 3 / (1+2e_4) \qquad (3.7.40)$$

$$CC = 3(1+2e_4) / (1-e_4)^3 \qquad (3.7.41)$$

$$e_5 + e_4 = 2/p_o^* \tag{3.7.42}$$

By using the equations (3.7.11) and (3.7.18) or (3.7.26) the bending moment distribution will be as shown in Fig. 3.7.1(b).

3.8 CONCLUSION

The graph is drawn for the collapse pressure p_o^* vs shell parameter CC as shown in Fig. 3.8.1 for various values of α and β. From this graph it is clear that e_2 and e_5 are always greater than β and e_1 and e_4 are always less than β, for β less than or equal to 0.5 and α upto 0.75. Hence the curves are smooth and only changes in slope are noticed at critical points. From this figure it is also clear that the negative circumferential stress is only confined upto 10 percent length of the top portion of the tank. For any given tank the collapse pressure and corresponding mode of failure can be determined by calculating the shell parameter CC.

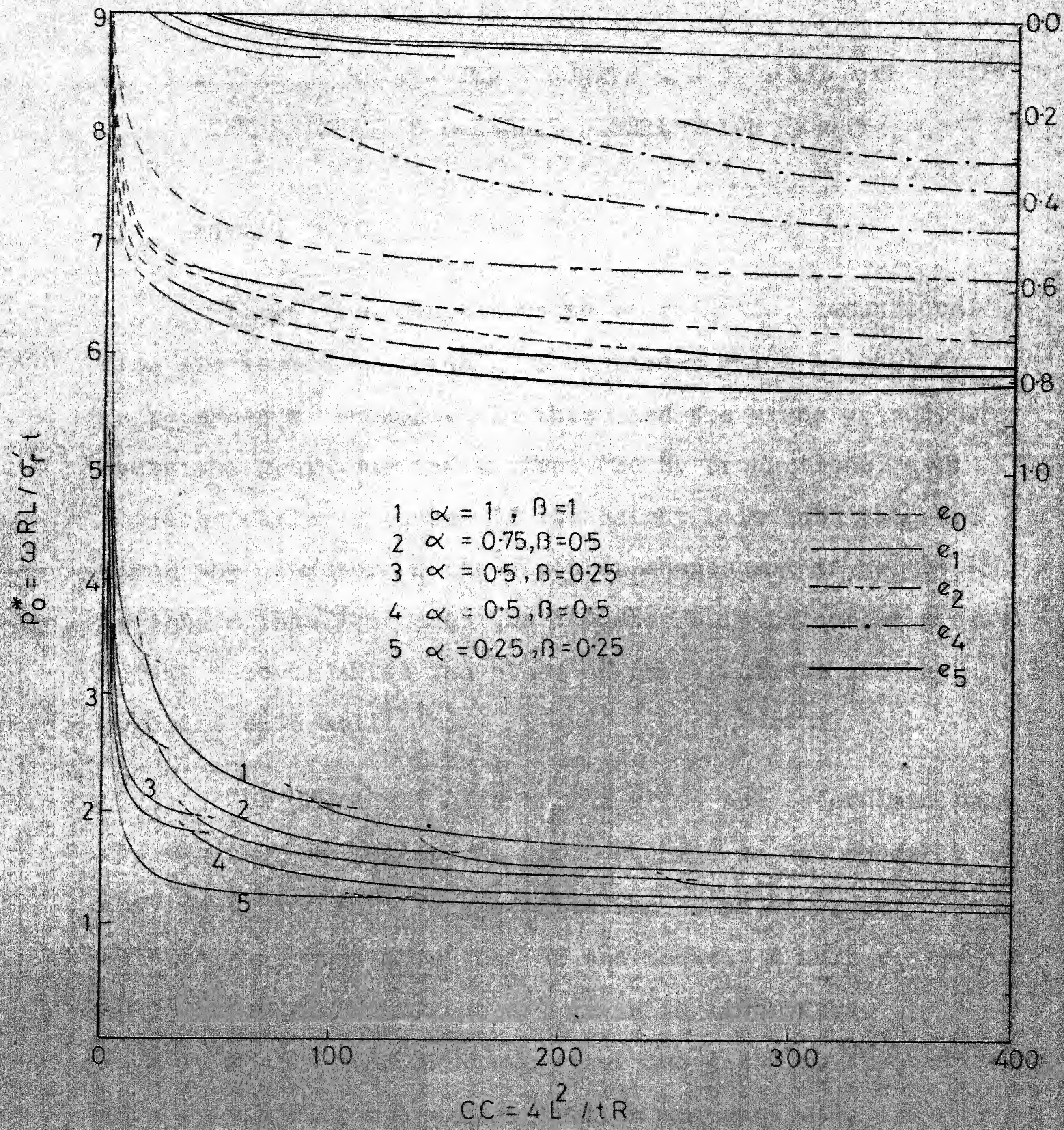


Fig. 3·8·1 p_o^* Vs CC for different α and β for tank

4. ANALYSIS OF REINFORCED CONCRETE SILOS BASED ON THE REIMBERT'S PRESSURE DISTRIBUTION THEORY

4.1 GENERAL

Large size containers to store grain, cement, coal etc. are termed as 'bins'. A container which is shallow is known as a 'bunker'. In this case the plane of rupture meets the grain surface at top. So by proportions, a bin would be called a bunker if its height L is less than 1.5 times the diameter D for circular shapes and if height L is larger than 1.5 times the diameter D it is called a silo. In the case of silos the plane of rupture meets at the opposite side wall(11).

The shape and size of the silos are determined by the engineer by taking the site, storage or any other service condition. In the silos the width of crack allowable is more than that of the tanks. A number of pressure distributions are given in literature(12). In this thesis, cylinderical silos subjected to Reimbert's and Janssen's pressure distribution are analysed. In this chapter plastic analysis of cylindrical silos subjected to Reimbert's pressure distribution is presented.

4.2 PRESSURE DISTRIBUTION

The material filled in cylindrical silos, in which emptying hole situated at the centre of the bottom, is held in equilibrium by the reaction from the sloping bottom and the frictional force on the vertical wall, as shown in Fig. 4.2.1[13].

The lateral unit pressure at depth X from top is

$$p = p_{max} \left(1 - (1 + X/C)^{-2} \right) \tag{4.2.1}$$

For circular silos

$$p_{max} = \gamma D/(4\mu') \tag{4.2.2}$$

and $$C = D / (4\mu' k) - h_s/3 \tag{4.2.3}$$

where

μ' = tan ϕ' , coefficient of friction between stored material and silo walls, and

k = $(1-\sin \phi)/ (1+\sin \phi)$

ϕ' = angle of friction between material stored and silo wall,

h_s = depth of the heap,

ϕ = angle of internal friction of stored material,

D = diameter of silo,

R_1 = hydraulic radius , and

γ = unit weight of stored material.

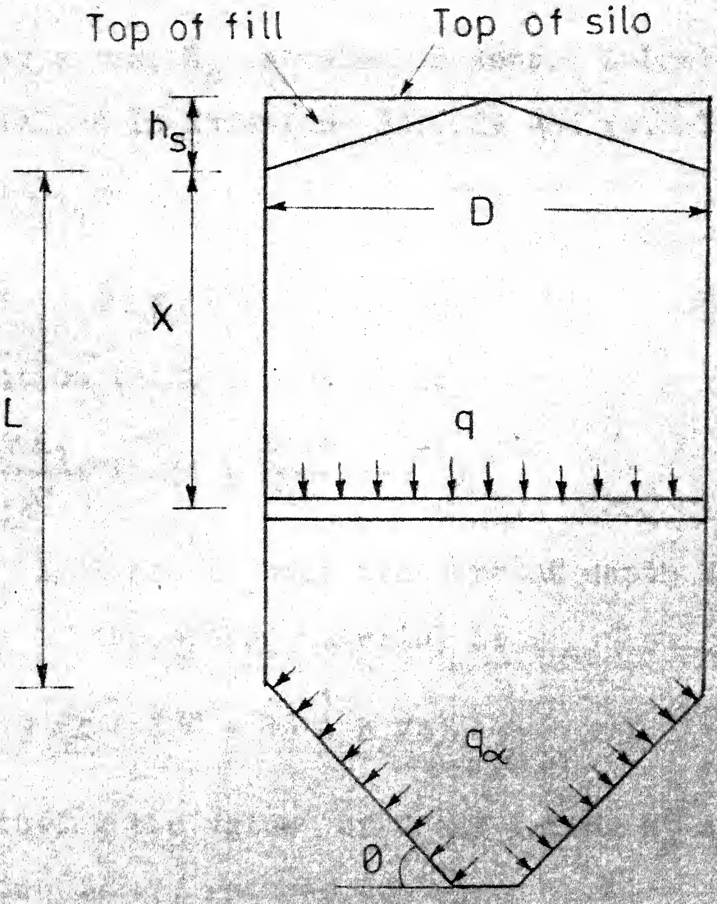


Fig.4·2·1 A typical silo and pressure distribution

For deep silos, h_s is taken as zero. Substituting these values in equations (4.2.2) and (4.2.3) , one gets,

$$p_{max} = \gamma R_1 / \mu' \tag{4.2.4}$$

$$C = R_1 / \mu' k \tag{4.2.5}$$

So equation (4.2.1) reduces to

$$p = \frac{\gamma R_1}{\mu'} \left(1 - \left(1 + \frac{X\mu' k}{R_1}\right)^{-2}\right) \tag{4.2.6}$$

The vertical static unit pressure at depth X below the surface of the stored material is

$$q = \gamma \left(X \left(X/C + 1\right)^{-1} + h_s/3\right) \tag{4.2.7}$$

Substituting the value of C and taking h_s as zero, equation (4.2.7) reduces to

$$q = \gamma \left(X \left(X\mu' k/R_1 + 1\right)^{-1}\right) \tag{4.2.8}$$

The vertical frictional force on a unit width of wall at depth X is

$$V = (\gamma X - q) R_1 \tag{4.2.9}$$

Static unit pressure normal to a surface inclined at an angle θ to the horizontal at depth X below surface of stored material is[12]

$$q_\alpha = p \sin^2\theta + q \cos^2\theta ; \tag{4.2.10}$$

4.3 EQUILIBRIUM EQUATIONS

The equilibrium equations for a silo in non-dimensional form are given by the equations (2.3.5) and (2.3.6). Unlike water tanks there is a vertical pressure along the walls besides the normal horizontal. The equilibrium equation for the normal pressure is given by

$$\frac{tR}{4L^2}\frac{d^2m_x}{de^2} + n_\theta = p^* \qquad (4.3.1)$$

where

$$\begin{aligned} p^* &= \frac{2\gamma R_1^2}{\mu'\sigma_r' t}\left(1-\left(1+\frac{X}{L}\,\frac{L\mu' k}{R_1}\right)^{-2}\right) \\ &= \frac{2\gamma R_1^2}{\mu'\sigma_r' t}\left(1-(1+eT)^{-2}\right) \\ &= p_o^*\left(1-(1+eT)^{-2}\right) \qquad (4.3.2) \end{aligned}$$

$$p_o^* = \frac{2\gamma R_1^2}{\mu'\sigma_r' t} \qquad (4.3.3)$$

$$T = \frac{\mu' kL}{R_1} \qquad (4.3.4)$$

$$CC = \frac{4L^2}{tR}, \text{ and} \qquad (4.3.5)$$

$$e = X/L \qquad (4.3.6)$$

where

γ = unit weight of stored material,

R_1 = hydraulic radius ,

μ' = coefficient of friction between stored material and silo walls,

T = nondimensional friction factor,

σ'_r = crushing stress of concrete in compression,

t = thickness of the shell wall,

CC = non-dimensional geometrical parameter of shells,

X = depth from top and

L = total depth of silos.

Substituting equations (4.3.2) and (4.3.5) in equation (4.3.1), one gets

$$\frac{d^2 m_x}{de^2} + CC\, n_\theta = CC\, p_o^* \left(1 - (1 + eT)^{-2} \right) \qquad (4.3.7)$$

4.4 SHORT SILO

4.4.1 Stress Distribution

A shallow silo is also known as a short silo. The failure of the silo at collapse is known as mode one failure as in tank. The distribution of the stress resultants and the collapse mechanism are shown in Fig. 4.4.1. The distribution of n_θ is as shown in Fig. 4.4.1(c). This is given by the equations

Zone I : $n_\theta = \alpha n_o$ for $0 \leq e \leq \beta$

Zone II : $n_\theta = \alpha n_o + K n_o (e-\beta)$ for $\beta \leq e \leq 1$ (4.4.1)

where

$$K = \frac{1-\alpha}{1-\beta}$$

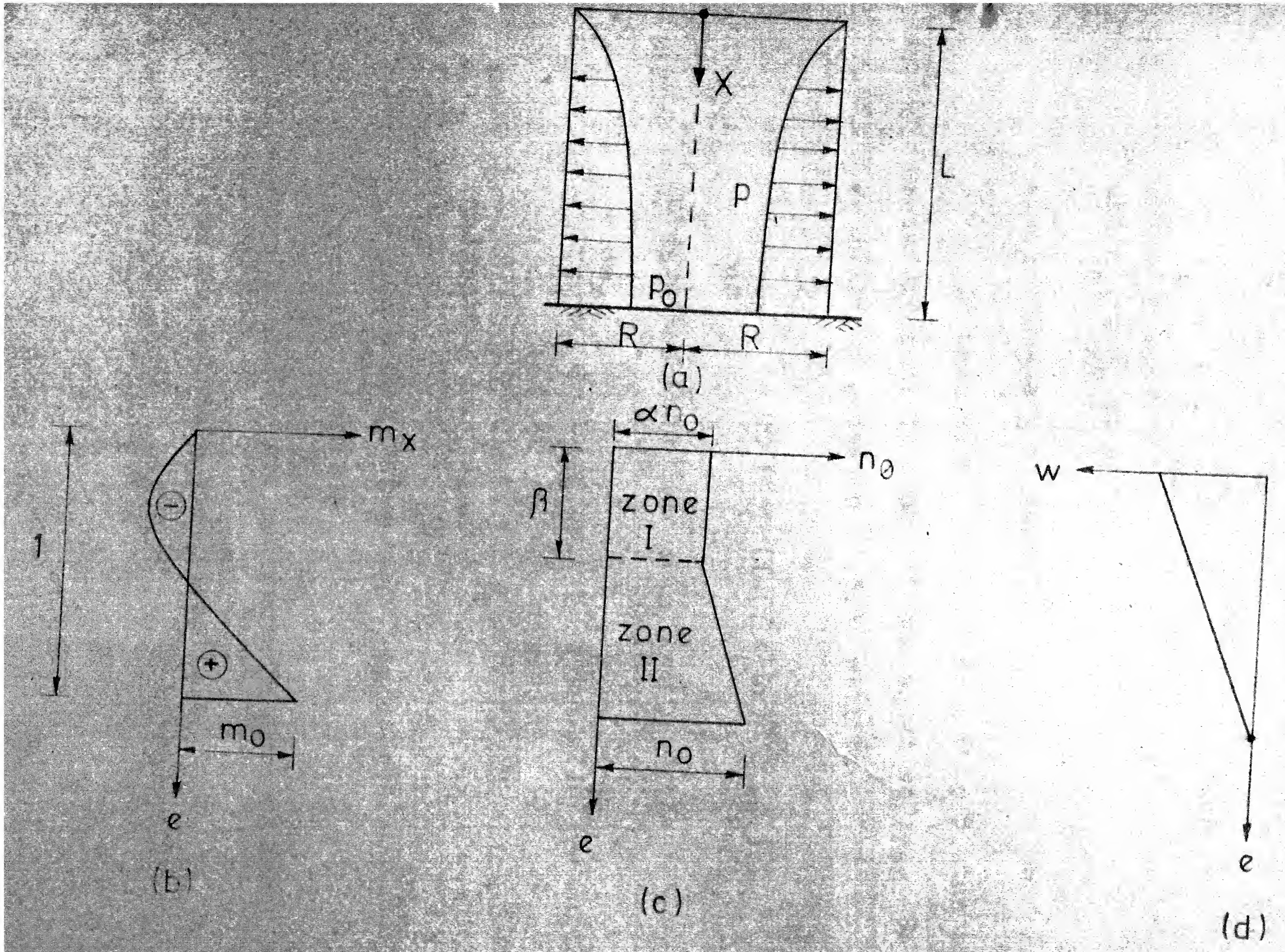


Fig.4·4·1 Stress distribution and collapse mechanism for short silo

The boundary and the continuity conditions are

$$m_{x1} = 0 \quad , \quad \frac{dm_{x1}}{de} = 0 \quad \text{at} \quad e = 0 \tag{4.4.2}$$

$$m_{x1} = m_{x2} \; , \; \frac{dm_{x1}}{de} = \frac{dm_{x2}}{de} \quad \text{at } e = \beta \tag{4.4.3}$$

and $m_{x1} = m_o$ at $e = 1$ (4.4.4)

4.4.2 $0 \leq e \leq \beta$ (Zone I)

In this zone, which is from 0 to β, the circumferential reinforcement is constant and equal to αn_o. So the equilibrium equation (4.3.5) will be

$$\frac{d^2 m_{x1}}{de^2} = CC \; (\; p_o^* \; (1-(1 + eT)^{-2} - \alpha n_o) \tag{4.4.5}$$

Integrating once

$$\frac{dm_{x1}}{de} = CC \; (p_o^* \; (e + T^{-1} (\; 1 + eT)^{-1}) - \alpha n_o e) + A_1 \tag{4.4.6}$$

Integrating once again

$$m_{x1} = CC \; (p_o^* \; (\frac{e^2}{2} + T^{-2} \ln(1+eT)^{-1}) - \frac{\alpha n_o e^2}{2}) + A_1 e + B_1 \tag{4.4.7}$$

where A_1 and B_1 are constants of integration.

From the boundary conditions (4.4.2)

$$A_1 = - \, CC \; p_o^* \; T^{-1} \tag{4.4.8}$$

$$B_1 = 0 \tag{4.4.9}$$

So equations (4.4.6) and (4.4.7) reduce to

$$\frac{dm_{x1}}{de} = CC\ (p_o^*(e+T^{-1}(1+eT)^{-1} - T^{-1}) - \alpha n_o e) \tag{4.4.10}$$

and

$$m_{x1} = CC\ (p_o^*(\ e + T^{-2}\ ln(1+eT) - T^{-1}e) - \frac{\alpha n_o e^2}{2}) \tag{4.4.11}$$

4.4.3 $\beta \leq e \leq 1$ (Zone II)

In this zone, which is from β to 1, the circumferential reinforcement is varying linearly from αn_o to n_o. So the equilibrium equation (4.3.5) becomes

$$\frac{d^3 m_{x2}}{de^2} = CC\ (p_o^*\ (1 - (1 + eT)^{-2}) - \alpha n_o - Kn_o(e-\beta)) \tag{4.4.12}$$

First integration gives

$$\frac{dm_{x2}}{de} = CC\ (p_o^*\ (e + T^{-1}(1+eT)^{-1}) - \alpha n_o e - Kn_o(\frac{e^2}{2} - \beta e)) + A_2 \tag{4.4.13}$$

Second integration gives

$$m_{x2} = CC(p_o^*(\frac{e^2}{2} + T^{-2} ln(1+eT)) - \frac{\alpha n_o e^2}{2} - Kn_o(\frac{e^3}{6} - \frac{\beta e^2}{2})) + A_2 e + B_2 \tag{4.4.14}$$

where A_2 and B_2 are constants of integration.

From the continuity conditions (4.4.3), one gets

$$A_2 = -CC\left(p_o^* T^{-1} + \frac{\beta^2 Kn_o}{2}\right) \tag{4.4.15}$$

$$B_2 = \frac{CC\ \beta^3\ Kn_o}{6} \tag{4.4.16}$$

Substituting the values of A_2 and B_2 in equations (4.4.13) and (4.4.14) , one gets

$$\frac{dm_{x2}}{de} = CC\left(p_o^*(e+T^{-1}(1+eT)^{-1}-T^{-1})-\alpha n_o e - \frac{Kn_o}{2}(e-\beta)^2\right) \tag{4.4.17}$$

and

$$m_{x2} = CC\left(p_o^*\left(\frac{e^2}{2} + T^{-2}\ln(1+eT)-T^{-1}e\right)- \frac{\alpha n_o e^2}{2} - \frac{Kn_o}{6}(e-\beta)^3\right) \tag{4.4.18}$$

From boundary condition (4.4.4)

$$p_o^* = \frac{6m_o/CC + 3\alpha n_o + Kn_o(1-\beta)^3}{3+6T^{-2}\ln(1+T) - 6T^{-1}} \tag{4.4.19}$$

By using the equations (4.4.11) and (4.4.18) , the bending moment distribution will be as shown in Fig. 4.4.1(b).

4.4.4 Critical Condition

As the L/R ratio of the shell increases, the values of CC as well as T increase . If L/R ratio remains same, but both L and R increase the value of CC increases and T remains constant. As CC increases p_o^* decreases, as is evident

from equation (4.4.19). As p_o^* decreases, m_{x1} and m_{x2} will also decrease which is evident from equations (4.4.11) and (4.4.18). But for certain value of CC = CCR and a particular $e = e_o$, m_{x1} or m_{x2} will reach the maximum negative value

$$m_x = -\bar{m}_o \quad \text{and} \quad \frac{dm_{x1}}{de} = 0 \tag{4.4.20}$$

when $e_o \leq \beta$

$$p_o^* = \frac{\alpha n_o e_o}{e_o + T^{-1}(1+e_oT)^{-1} - T^{-1}} \tag{4.4.21}$$

and

$$CCR = \frac{2\bar{m}_o}{\alpha n_o e_o^2 - p_o^*(e_o^2 + 2T^{-2}\ln(1+e_oT) - 2T^{-1}e_o)} \tag{4.4.22}$$

when $\beta \leq e_o \leq 1$

$$p_o^* = \frac{2\alpha n_o e_o + Kn_o(e_o-\beta)^2}{2e_o + 2T^{-1}(1+e_oT)^{-1} - 2T^{-1}} \tag{4.4.23}$$

and

$$CCR = \frac{6\bar{m}_o}{(3\alpha n_o e_o^2 + Kn_o(e_o-\beta)^3 - p_o^*(3e_o^2 + 6T^{-2}\ln(1+e_oT) - 6T^{-1}e_o))} \tag{4.4.24}$$

4.4.5 Special Case

Let $m_o = \bar{m}_o = n_o = 1$

Then

$$p_o^* = \frac{6/CC + 3\alpha + K(1-\beta)^3}{3(1+2T^{-2}\ln(1+T)-2T^{-1})} \qquad (4.4.25)$$

when $e_o \leq \beta$

$$p_o^* = \frac{\alpha(1+e_oT)}{e_oT} \qquad (4.4.26)$$

and

$$CCR = \frac{2}{\alpha e_o^2 - p_o^*(e_o^2+2T^{-2}\ln(1+e_oT)-2T^{-1}e_o)} \qquad (4.4.27)$$

when $\beta \leq e_o \leq 1$

$$p_o^* = \frac{2\alpha e_o + K(e_o - \beta)^2}{2(e_o + T^{-1}(1+e_oT)^{-1} - T^{-1}} \qquad (4.4.28)$$

and

$$CCR = \frac{6}{(3\alpha e_o^2 + K(e_o-\beta)^3 - 3p_o^*(e_o^2+2T^{-2}\ln(1+e_oT)-2T^{-1}e_o)} \qquad (4.4.29)$$

when $\alpha = \beta = 1$

$$p_o^* = \frac{2}{CC(1+2T^{-2}\ln(1+T)-2T^{-1})} \qquad (4.4.30)$$

$$e_o = \frac{1}{T(p_o^* - 1)} \qquad (4.4.31)$$

$$CCR = \frac{2}{e_o^2 - p_o^*(e_o^2+2T^{-2}\ln(1+e_oT)-2T^{-1}e_o)} \qquad (4.4.32)$$

4.4.6 Results

The distribution of n_θ is dependent upon the values of α and β. For a given set of α and β values and for prescribed T, the critical shell parameter value CCR, the location of hinge e_o, and the collapse pressure p_o^* are shown in Table 4.1, 4.2 and 4.3 for $m_o = \bar{m} = n_o = 1$ and for T equal to 1, 2.5 and 4 respectively.

TABLE 4.1 : CCR FOR FIRST MODE OF FAILURE OF SILOS AT T=1 AND $m_o = \bar{m}_o = n_o = 1$

α	β	CCR	e_o	p_o^*
0.25	0.25	267.5	0.338	1.0300
0.50	0.25	66.7	0.506	1.6140
0.50	0.50	58.5	0.504	1.4897
0.75	0.50	35.0	0.539	2.1463
1.00	1.00	24.6	0.549	2.7490

TABLE 4.2 : CCR FOR FIRST MODE OF FAILURE OF SILOS AT $T = 2.5$ AND $m_o = \bar{m}_o = n_o = 1$

α	β	CCR	e_o	p_o^*
0.25	0.25	515.0	0.249	0.656
0.50	0.25	111.8	0.424	1.018
0.50	0.50	88.5	0.455	0.939
0.75	0.50	50.2	0.500	1.350
1.00	1.00	34.8	0.518	1.760

TABLE 4.3 : CCR FOR FIRST MODE OF FAILURE OF SILOS AT $T=4$ AND $m_o=\bar{m}_o=n_o=1$

α	β	CCR	e_o	p_o^*
0.25	0.25	819.0	0.201	0.560
0.50	0.25	163.0	0.362	0.864
0.50	0.50	119.0	0.421	0.796
0.75	0.50	65.0	0.477	1.144
1.00	1.00	44.2	0.494	1.490

From the forgoing Tables 4.1 to 4.3 , it is clear that as the value of T increases the range of CCR for first mode increases. The increase in T corresponds to the rise in rigidity of the silo.

There is one point to be noted in these tables that for $\alpha = 0.5$, $\beta = 0.25$ and $\alpha = 0.5$, $\beta = 0.5$, CCR and p_o^* is more for first case, but e_o is less for second case.

4.5 MEDIUM SILO

4.5.1 Stress Distribution

Failure in this silo will be same as medium tank, that is mode two failure.

The stress distribution and collapse mechanism is as shown in Fig. 4.5.1.

The distribution of n_θ is as shown in Fig. 4.5.1(b) . There are three zones

Zone I : $n_\theta = -\alpha \bar{n}_o$ for $0 \leq e \leq e_1$

Zone II : $n_\theta = \alpha n_o$ for $e_1 \leq e \leq \beta$

Zone III : $n_\theta = \alpha n_o + K(e-\beta)\, n_o$ for $\beta \leq e \leq 1$ (4.5.1)

The boundary and the continuity conditions are

$$m_{x1} = 0 \ , \ \frac{dm_{x1}}{de} = 0 \text{ at } e = 0 \qquad (4.5.2)$$

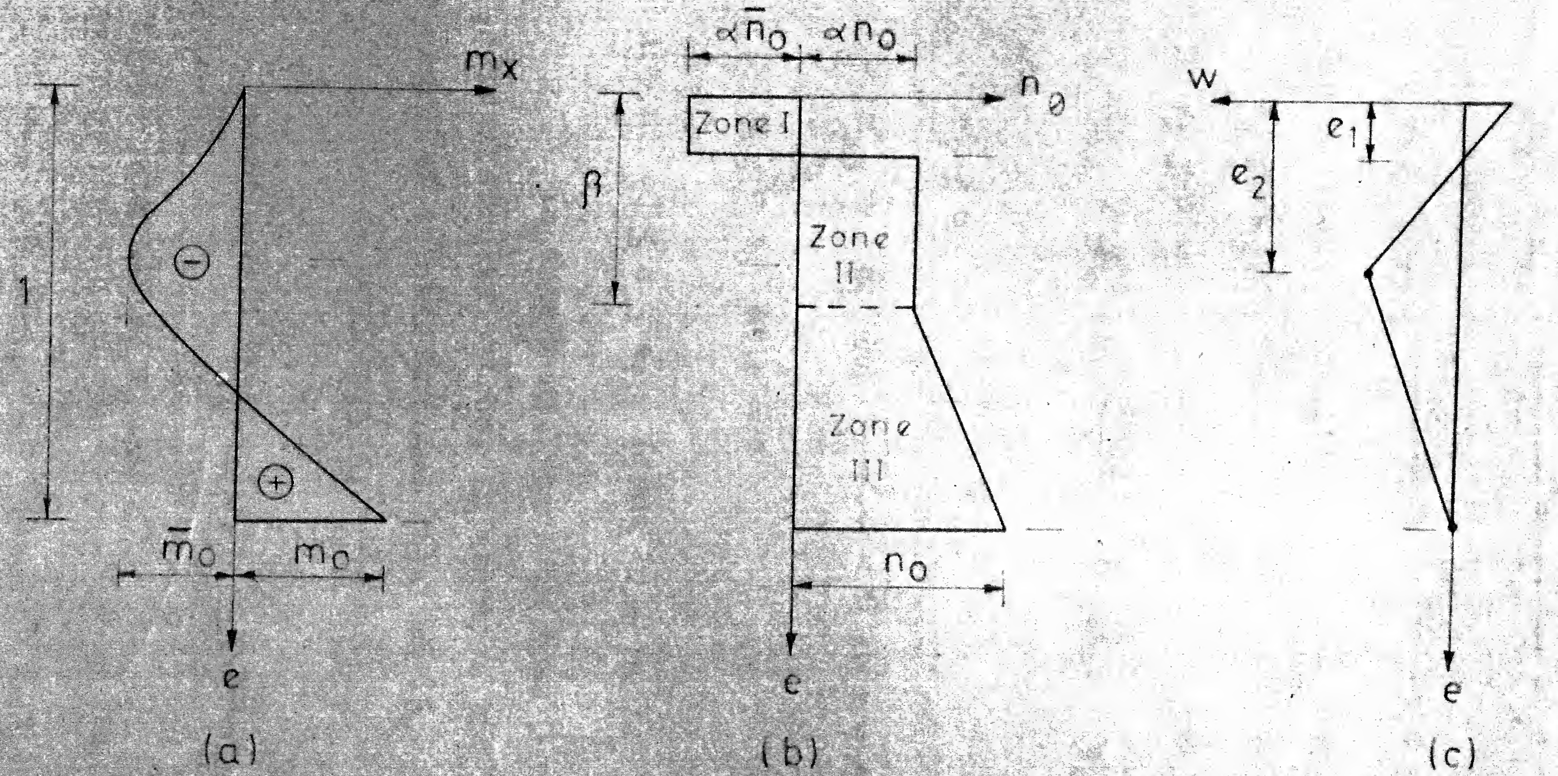


Fig. 4·5·1 Stress distribution and collapse mechanism for medium silo

$$m_{x1} = m_{x2}, \quad \frac{dm_{x2}}{de} = \frac{dm_{x2}}{de} \quad \text{at } e = e_1 \tag{4.5.3}$$

$$m_{x2} = m_{x3}, \quad \frac{dm_{x2}}{de} = \frac{dm_{x3}}{de} \quad \text{at } e = \beta \tag{4.5.4}$$

The continuity condition at $e = e_2$ depends on whether

$e_2 \gtrless \beta$

If $e_2 \leq \beta$

$$m_{x2} = -\bar{m}_o \text{ and } \frac{dm_{x2}}{de} = 0 \quad \text{at } e = e_2$$

and

$e_2 \geq \beta$

$$m_{x3} = -\bar{m}_o \text{ and } \frac{dm_{x3}}{de} = 0 \text{ at } e = e_2 \tag{4.5.5}$$

$$m_{x3} = m_o \quad \text{at } e = 1 \tag{4.5.6}$$

4.5.2 $\quad 0 \leq e \leq e_1 \quad$ (Zone I)

In this zone, which is from 0 to e_1, the circumferential stress is negative and reinforcement provided is constant and equal to $\alpha\bar{n}_o$. So the equilibrium equation (4.3.5) will be

$$\frac{d^2 m_{x1}}{de^2} = CC \left(p_o^*(1-(1+eT)^{-2}) + \alpha\bar{n}_o\right) \tag{4.5.7}$$

First integration gives

$$\frac{dm_{x1}}{de} = CC \left(p_o^*(e+T^{-1}(1+eT)^{-1})+\alpha\bar{n}_o e\right) + A_1 \tag{4.5.8}$$

Second integration gives

$$m_{x1} = CC(p_o^* (\frac{e^2}{2} + T^{-2}\ln(1+eT)) + \frac{\alpha\bar{n}_o e^2}{2}) + A_1 e + B_1 \tag{4.5.9}$$

where A_1 and B_1 are constants of integration.

From boundary conditions (4.5.2), one gets

$$A_1 = - CC\ p_o^*\ T^{-1} \tag{4.5.10}$$

$$B_1 = 0 \tag{4.5.11}$$

So equations (4.5.8) and (4.5.9) will be

$$\frac{dm_{x1}}{de} = CC\ (p_o^* (e+T^{-1}(1+eT)^{-1}-T^{-1})+\alpha\bar{n}_o e) \tag{4.5.12}$$

and

$$m_{x1} = CC\ (p_o^* (\frac{e^2}{2} + T^{-2}\ln\ (1+eT) - T^{-1}e)+ \frac{\alpha\bar{n}_o e^2}{2}) \tag{4.5.13}$$

4.5.3 $e_1 \leq e \leq \beta$ (Zone II)

In this zone, which is from e_1 to β, the circumferential stress is positive and reinforcement provided is constant, equal to αn_o, from e_1 to β. So the equilibrium equation (4.3.5) will be

$$\frac{d^2m_{x2}}{de^2} = CC\ (p_o^*(1-(1+eT)^{-2}) -\alpha n_o) \tag{4.5.14}$$

First integration gives

$$\frac{dm_{x2}}{de} = CC\ (p_o^*(e+T^{-1}(1+eT)^{-1}) - e\alpha n_o) + A_2 \qquad (4.5.15)$$

Second integration gives

$$m_{x2} = CC\ (p_o^*\ (\frac{e^2}{2} + T^{-2}\ln(1+eT)) - \frac{\alpha n_o e^2}{2}) + A_2 e + B_2 \qquad (4.5.16)$$

where A_2 and B_2 are constants of integration.

Using the continuity conditions (4.5.3), one gets

$$A_2 = CC\ (\alpha e_1(n_o+\bar{n}_o)-p_o^* T^{-1}) \qquad (4.5.17)$$

$$B_2 = -\frac{CC\alpha e_1^2}{2}\ (n_o + \bar{n}_o) \qquad (4.5.18)$$

Substituting these values in equations (4.5.15) and (4.5.16), one gets

$$\frac{dm_{x2}}{de} = CC\ (p_o^*\ (e+T^{-1}(1+eT)^{-1}-T^{-1}) - \alpha n_o e + \alpha e_1\ (n_o+\bar{n}_o)) \qquad (4.5.19)$$

and

$$m_{x2} = CC\ (p_o^*\ (\frac{e^2}{2} + T^{-2}\ln(1+eT) - T^{-1}e) - \frac{\alpha n_o e^2}{2} + (\alpha e_1 e - \frac{\alpha e_1^2}{2})\ (n_o+\bar{n}_o)) \qquad (4.5.20)$$

4.5.4 $\beta \leq e \leq 1$ (Zone III)

In this zone, which is from β to 1, the circumferential stress is positive and reinforcement provided is linearly

varying from αn_o to n_o from β to 1. So the equilibrium equation (4.3.5) will reduces to

$$\frac{d^2 m_{x3}}{de^2} = CC\ (p_o^*(1-(1+eT)^{-2})-\alpha n_o - Kn_o(e-\beta)) \qquad (4.5.21)$$

By first integration

$$\frac{dm_{x3}}{de} = CC\ (p_o^*(e+T^{-1}(1+eT)^{-1})-\alpha n_o e - Kn_o(\frac{e^2}{2} - \beta e)) + A_3 \qquad (4.5.22)$$

By second integration

$$m_{x3} = CC\ (p_o^*(\frac{e^2}{2} + T^{-2}\ln(1+eT)) - \frac{\alpha n_o e^2}{2} - Kn_o(\frac{e^3}{6} - \frac{\beta e^2}{2})) + A_3 e + B_3 \qquad (4.5.23)$$

where A_3 and B_3 are constants of integration.

From the continuity condition (4.5.4) .

$$A_3 = CC\ (\alpha e_1 (n_o+\bar{n}_o) - n_o K\beta^2/2 - p_o^*\ T^{-1}) \qquad (4.5.24)$$

and

$$B_3 = CC\ (\frac{Kn_o\beta^3}{6} - \frac{\alpha e_1^2}{2}(n_o+\bar{n}_o)) \qquad (4.5.25)$$

Substituting these values in equations (4.5.22) and (4.5.23), one gets

$$\frac{dm_{x3}}{de} = CC(p_o^*(e+T^{-1}(1+eT)^{-1}-T^{-1})-n_o\alpha e_1+\alpha e_1(n_o+\bar{n}_o) - \frac{n_o K}{2}(e-\beta)^2)) \qquad (4.5.26)$$

and

$$m_{x3} = CC(p_o^*(\frac{e^2}{2} + T^{-2}\ln(1+eT)-T^{-1}e) - \frac{n_o\alpha e^2}{2}$$

$$+ \alpha e_1 e(n_o+\bar{n}_o) - \frac{n_oK}{6}(e-\beta)^3) \qquad (4.5.27)$$

Using boundary condition (4.5.6) in equation (4.5.27) one gets

$$p_o^* = \frac{(6m_o/CC + 3\alpha n_o+n_oK(1-\beta)^3-\alpha(n_o+\bar{n}_o)(6e_1-3e_1^2))}{3(1+2T^{-2}\ln(1+T) \quad -2T^{-1})} \qquad (4.5.28)$$

From the first condition of (4.5.5)

$$p_o^* = \frac{\alpha n_o e_2-\alpha e_1 \ (n_o+\bar{n}_o)}{e_2+T^{-1}(1+e_2T)^{-1}-T^{-1}} \qquad (4.5.29)$$

$$CC = \frac{2m_o}{(\alpha n_o e_2^2-(2\alpha e_1 e_2-\alpha e_1^2) \quad (n_o + \bar{n}_o \ ) -p_o^*(e_2^2+2T^{-2}\ln \ (1+e_2T)-2T^{-1}e_2))} \qquad (4.5.30)$$

From t e second condition of (4.5.5)

$$p_o^* = \frac{2n_o\alpha e_2 + n_oK(e_2-\beta)^2-2\alpha e_1(n_o+\bar{n}_o)}{2(e_2+T^{-1}(1+e_2T)^{-1}-T^{-1})}$$

and

$$CC = \frac{6\bar{m}_o}{(3n_o\alpha e_2^2+3\alpha(n_o+\bar{n}_o)(e_1^2-2e_1e_2)-n_oK(e_2-\beta)^3 -3p_o^*(e_2^2+2T^{-2}\ln(1+e_2T)-2T^{-1}e_2))} \qquad (4.5.32)$$

By using equations (4.5.13), (4.5.20) and (4.5.27), the bending moment distribution will be as shown in Fig. 4.5.1(a).

4.5.5 Critical Condition

As the value of CC will increase the positive moment above the negative hinge will increase, and it will reach to positive maximum moment m_o for certain value of CC equal to CCR at $e = e_3$. This is clear from equation (4.5.13). The value of CCR will also depend on whether $e_3 \gtrless \beta$. e_1 is confined upto 0.1 L of the upper portion, so there is no point in considering $\beta < e_1$.

So at $e = e_3$.

$$m_x = m_o \text{ and } \frac{dm_x}{de} = 0 \qquad (4.5.33)$$

when

$$e_3 \leq \beta$$

$$p_o^* = \frac{\alpha n_o(1+e_2T)\,(1+e_3T)}{(e_2T+e_3T + e_2e_3T^2)} \qquad (4.5.34)$$

and

$$CCR = \frac{2m_o}{(p_o^*(e_3^2+2T^{-2}\ln(1+e_3T)-2T^{-1}e_3)-\alpha n_o e_3^2+(n_o+\bar{n}_o)(2\alpha e_1e_2-2e_1^2))} \qquad (4.5.35)$$

and when $e_3 \geq \beta$

$$p_o^* = \frac{(\alpha n_o + n_oK/2\,(e_2+e_3)-n_oK\beta)(1+e_2T)(1+e_3T)}{(e_2T+e_3T + T^2e_2e_3)} \qquad (4.5.36)$$

and

$$CCR = \frac{6m_o}{(3p_o^*(e_3^2+2T^{-2}\ln(1+e_3T)-2T^{-1}e_3) - 3n_o\alpha e_3^2 + \alpha(n_o+\bar{n}_o)\ (6e_1e_3-3e_1^2)-n_oK(e_3-\beta)^3)} \qquad (4.5.37)$$

4.5.6 Special Case

Let $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

Then equation (4.5.28) reduces to

$$p_o^* = \frac{6/CC + 3\alpha-6\alpha(2e_1-e_1^2)+K\ (1-\beta)^3}{3(1+2T^{-2}\ln(1+T)-2T^{-1})} \qquad (4.5.38)$$

When $e_2 \leq \beta$, equations (4.5.29) and (4.5.30) will be

$$p_o^* = \frac{\alpha e_2 - 2\alpha e_1}{(e_2 + T^{-1}(1+e_2T)^{-1}-1)} \qquad (4.5.39)$$

$$CC = \frac{2}{(\alpha e_2^2-4\alpha e_1e_2+2\alpha e_1^2-p_o^*(e_2^2+2T^{-2}\ln(1+e_2T)-2T^{-1}e_2\ ))} \qquad (4.5.40)$$

when $e_2 \geq \beta$, equations (4.5.31) and (4.5.32) will be

$$p_o^* = \frac{2\alpha e_2 + K(e_2-\beta)^2-4\alpha e_1}{2(e_3+T^{-1}(1+e_2T)^{-1} - T^{-1}\ )} \qquad (4.5.41)$$

and

$$CC = \frac{6}{(3\alpha e_2^2+6\alpha e_1^2 + K(e_2-\beta)^3-12\alpha e_1e_2 - p_o^*(3e_2^2+6T^{-2}\ln(e_2T+1) - 6T^{-1}e_2))} \qquad (4.5.42)$$

when $e_3 \leq \beta$, equations (4.5.34) and (4.5.35) will be

$$p_o^* = \alpha(1+e_2T)\ (1+e_3T)\ /\ (e_2T+e_3T+e_2e_3T^2) \tag{4.5.43}$$

and

$$CCR = \frac{2}{p_o^*(e_3^2+2T^{-2}\ln(1+e_3T)-2T^{-1}e_3)-\alpha n_o e_3^2+2(2\alpha e_1 e_2-\alpha e_1^2)} \tag{4.5.44}$$

when $e_3 \geq \beta$, equations (4.5.36) and (4.5.37) will be

$$p_o^* = \frac{((\alpha+K/2(e_2+e_3)-K\beta)(1+e_2T)(1+e_3T))}{T(e_2 + e_3 + e_2e_3T)} \tag{4.5.45}$$

and

$$CCR = \frac{6}{p_o^*(3e_3^2+6T^{-2}\ln(1+e_3T)-6T^{-1}e_3-3\alpha e_3^2 -K(e_3-\beta)^3+6\alpha(3e_1e_3-e_1^2))} \tag{4.5.46}$$

when $\alpha = \beta = 1$

$$p_o^* = (6/CC+3-6e_1(2-e_1))/\ 3(1+2T^{-2}\ln(1+T)-2T^{-1}) \tag{4.5.47}$$

$$p_o^* = (e_2-2e_1)/(e_2+T^{-1}(1+e_2T)^{-1}-1) \tag{4.5.48}$$

$$CC = 2/(e_2^2-2(2e_1e_2-e_1^2)-p_o^*(e_2^2+2T^{-2}\ln(1+e_2T)-2T^{-1}e_2)) \tag{5.4.49}$$

$$p_o^* = (1+e_2T)(1+e_3T)/(e_2T+e_3T+e_2e_3T^2) \qquad (4.5.50)$$

$$CCR = 2/(p_o^*(e_3^2+2T^{-2}\ln(1+e_3T)-2T^{-1}e_3)) \qquad (4.5.51)$$

4.5.7 Critical Values

The distribution of n_θ is dependent upon the values of α and β. For a given set of α and β values at certain T the critical shell parameter value CCR, location of hinges e_2 and e_3, the range of hoop compression from 0 to e_1, and collapse pressure p_o^* are shown in Table 4.4 for $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$ and $T = 1$.

TABLE 4.4 : CCR FOR SECOND MODE OF FAILURE OF SILOS AT $T = 1$ AND $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

α	β	CCR	e_1	e_2	e_3	p_o^*
0.25	0.25	-	-	-	-	-
0.50	0.25	-	-	-	-	-
0.50	0.50	-	-	-	-	-
0.75	0.50	-	-	-	-	-
1.00	1.00	197.2	0.061	0.64	0.214	2.006

From Table 4.4 it is clear that the CCR for second mode, for α and β less than one, is more than 400. So the range of second mode is large and as the value of T increases this range will also increase.

4.6 DEEP SILO

4.6.1 Stress Distribution

Failure in this mode is known as mode three failure and also known as partial failure or partial collapse because in this failure upper portion remains intact as in the third mode of failure in tank. The stress distribution and collapse mechanism are as shown in Fig. 4.6.1. The distribution of n_θ is as shown in Fig. 4.6.1(b). This is given by the equations.

$$\text{Zone I}: \quad n_\theta = \alpha n_o \text{ for } e_4 \leq e \leq \beta$$

$$\text{Zone II}: \quad n_\theta = \alpha n_o + K(e-\beta) n_o \text{ for } \beta \leq e \leq 1 \qquad (4.6.1)$$

The boundary and the continuity conditions are

$$m_{x1} = m_o, \quad \frac{dm_{x1}}{de} = 0 \text{ at } e = e_4 \text{ for } e_4 \leq \beta$$

$$m_{x2} = m_o, \quad \frac{dm_{x2}}{de} = 0 \text{ at } e = e_4 \text{ for } e_4 \geq \beta \qquad (4.6.2)$$

$$m_{x1} = m_{x2}, \quad \frac{dm_{x1}}{de} = \frac{dm_{x2}}{de} \text{ at } e = \beta \text{ for } e_4 \leq \beta \quad (4.6.3)$$

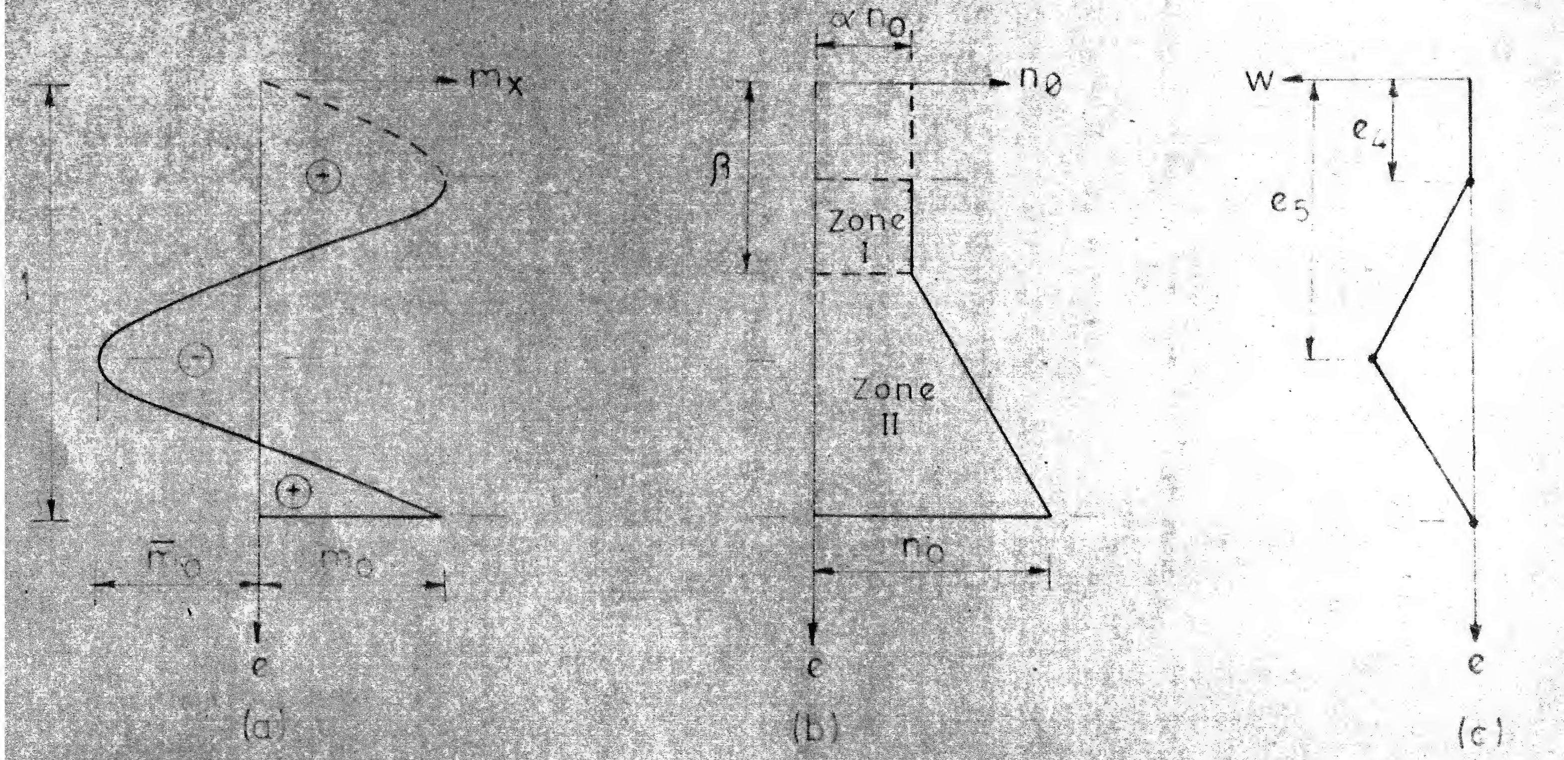


Fig. 4·6·1 Stress distribution and collapse mechanism for deep silo

The continuity conditions at $e = e_5$ depends on whether

$e_5 \gtrless \beta$

For $e_5 \leq \beta$

$$m_{x1} = -\bar{m}_o \ , \quad \frac{dm_{x1}}{de} = 0 \quad \text{at } e = e_5$$

and for $e_5 \geq \beta$

$$m_{x2} = -\bar{m}_o \ , \quad \frac{dm_{x2}}{de} = 0 \text{ at } e = e_5 \qquad (4.5.4)$$

and $m_{x2} = m_o$ at $e = 1$ (4.6.5)

4.6.2 $e_4 \leq e \leq \beta$ (Zone I)

In this zone, circumferential stress is positive and reinforcement is constant throughout, from e_4 to β, is αn_o. So equation (4.3.5) reduces to

$$\frac{d^2 m_{x1}}{de^2} = CC\ (p_o^*(1-(1+eT)^{-1}) - \alpha n_o) \qquad (4.6.6)$$

First integration gives

$$\frac{dm_{x1}}{de} = CC\ (p_o^*\ (e+T^{-1}(1+eT)^{-1})-\alpha n_o e) + A_4 \qquad (4.6.7)$$

Second integration gives

$$m_{x1} = CC\ (p_o^*(\frac{e^2}{2} + T^{-2}\ln(1+eT))- \frac{\alpha n_o e^2}{2}\ )+ A_4 e+B_4 \qquad (4.6.8)$$

where A_4 and B_4 are constants of integration.

From boundary conditions (4.6.2)

$$A_4 = -\,CC\,(p_o^*(e_4+T^{-1}(1+e_4T)^{-1})-\alpha n_o e_4)$$

$$B_4 = CC\,(p_o^*\,(\frac{e_4^2}{2} - T^{-2}\ln(1+e_4T)+T^{-1}e_4(1+e_4T)^{-1}) - \frac{\alpha n_o e_4^2}{2}) + m_o \qquad (4.6.9)$$

Substituting these values in equations (4.6.7) and (4.6.8), one gets

$$\frac{dm_{x1}}{de} = CC(p_o^*(e+T^{-1}(1+eT)^{-1}-e_4-T^{-1}(1+e_4T)^{-1}-\alpha n_o e+\alpha n_o e_4) \qquad (4.6.10)$$

and

$$m_{x1} = CC\,(p_o^*((e-e_4)^2/2 + T^{-2}(\ln(1+eT)-\ln(1+e_4T)) - T^{-1}(1+e_4T)^{-1}(e-e_4)) - \frac{\alpha n_o}{2}\,(e-e_4)^2) + m_o \qquad (4.6.11)$$

4.6.3 $\beta \leq e \leq 1$ (Zone II)

In this zone the circumferential stress is positive from β to 1 , and the reinforcement provided in this zone is linearly varying from αn_o to n_o. So the equation (4.3.5) will be

$$\frac{d^2 m_{x2}}{de^2} = CC\,(p_o^*(1-(1+eT)^{-2})-\alpha n_o-Kn_o(e-\beta)) \qquad (4.6.12)$$

Integrating once

$$\frac{dm_{x2}}{de} = CC\ (p_o^*(e+T^{-1}(1+eT)^{-1})-\alpha n_o e-Kn_o(e^2/2-\beta e))+A_5 \tag{4.6.13}$$

Integrating once again

$$m_{x2} = CC\ (p_o^*(e^2/2+T^{-2}\ln(1+eT))-\alpha n_o e^2/2-Kn_o(e^3/6-\beta e^2/2))$$
$$+A_5 e+B_5 \tag{4.6.14}$$

where A_5 and B_5 are constants of integration.

From the continuity conditions (4.6.3)

$$A_5 = CC\ (\alpha n_o e_4-p_o^*(e_4+T^{-1}(1+e_4T)^{-1})-Kn_o\beta^2/2) \tag{4.6.15}$$

and

$$B_5 = CC(p_o^*(e_4^2/2-T^{-2}\ln(1+e_4T)+e_4T^{-1}(1+e_4T)^{-1})$$
$$-\alpha n_o e_4^2/2+Kn_o\beta^3/6)+m_o \tag{4.6.16}$$

Substituting these values in equation (4.6.13) and (4.6.14) , one gets

$$\frac{dm_{x2}}{de} = CC(p_o^*(e+T^{-1}((1+eT)^{-1}-(1+e_4T)^{-1})-e_4)$$
$$-\alpha n_o(e-e_4)-Kn_o/2(e-\beta)^2) \tag{4.6.17}$$

and

$$m_{x2} = CC(p_o^*((e-e_4)^2/2+T^{-2}(\ln(1+eT)-\ln(1+e_4T))$$
$$-T^{-1}(1+e_4T)^{-1}(e-e_4))-\alpha n_o/2(e-e_4)^2-Kn_o/6(e-\beta)^3)+m_o \tag{4.6.18}$$

using the boundary condition (4.6.5) in equation (4.6.18)

$$p_o^* = \frac{3\alpha n_o(1-e_4)^2+Kn_o(1-\beta)^3}{3((1-e_4)^2+2T^{-2}(\ln(1+T)-\ln(1+e_4T))-2T^{-1}(1+e_4T)^{-1}(1-e_4))} \tag{4.6.19}$$

4.6.4 $e_4 \leq \beta$ (Case I)

For $e_5 \leq \beta$, using the continuity conditions (4.6.4) in equations (4.6.11)

$$p_o^* = \frac{\alpha n_o(e_5-e_4)}{(e_5-e_4) + T^{-1}((1+e_5T)^{-1}-(1+e_4T)^{-1})} \tag{4.6.20}$$

and

$$CC = \frac{2(m_o+\bar{m}_o)}{\alpha n_o(e_5-e_4)^2-p_o^*\left((e_5-e_4)^2+2T^{-2}(\ln(1+e_5T)-\ln(1+e_4T))-2T^{-1}(1+e_4T)^{-1}(e_5-e_4)\right)} \tag{4.6.21}$$

For $e_5 \geq \beta$, using the continuity conditions (4.6.4) in equations (4.6.17) and (4.6.18), one gets

$$p_o^* = \frac{2\alpha n_o(e_5-e_4) + Kn_o(e_5-\beta)^2}{2((e_5-e_4) + T^{-1}((1+e_5T)^{-1}-(1+e_4T)^{-1}))} \tag{4.6.22}$$

and

$$CC = \frac{6(m_o+\bar{m}_o)}{3\alpha n_o(e_5-e_4)^2+Kn_o(e_5-\beta)^3-p_o^*(3(e_5-e_4)^2+6T^{-2}(\ln(1+e_5T)-\ln(1+e_4T))-T^{-1}(1+e_4T)^{-1}(e_5-e_4))} \tag{4.6.23}$$

4.6.5 $\beta \leq e_4$ (Case II)

For this case equilibrium equation (4.3.5) is

$$\frac{d^2 m_{x3}}{de^2} = CC\ (p_o^*(1-(1+eT)^{-2}-\alpha n_o - Kn_o(e-\beta)) \qquad (4.6.24)$$

Integrating once

$$\frac{dm_{x3}}{de} = CC\ (p^*(e+T^{-1}(1+eT)^{-1})-\alpha n_o e - Kn_o(e^2/2-\beta e)) + A_6 \qquad (4.6.25)$$

Integrating once again

$$m_{x3} = CC\ (p_o^*(e^2/2+T^{-2}\ln(1+eT)) - \frac{\alpha n_o e^2}{2} - Kn_o(e^3/6-\beta e^2/2)) + A_6 e + B_6 \qquad (4.6.26)$$

Where A_6 and B_6 are constants of integration.

Using continuity conditions (4.6.2) , one gets

$$A_6 = -CC(p_o^*(e_4+T^{-1}(1+e_4T)^{-1}-\alpha n_o e_4 - Kn_o(e_4^2/2-\beta e_4)) \qquad (4.6.27)$$

and

$$B_6 = m_o - CC(p^*_o(T^{-2}\ln(1+e_4T)-e_4^2/2-T^{-1}e_4(1+e_4T)^{-1}) + \frac{\alpha n_o e_4^2}{2} + Kn_o(e_4^3/3-\beta e_4^2/2) \qquad (4.6.28)$$

Substituting these values in equation (4.6.25) and (4.6.26), one gets

$$\frac{dm_{x3}}{de} = CC(p_o^*(e+T^{-1}(1+eT)^{-1}-e_4-T^{-1}(1+e_4T)^{-1})-\alpha n_o(e-e_4)$$
$$-Kn_o(e^2/2 - \beta e - e_4^2/2 + \beta e_4)) \qquad (4.6.29)$$

and

$$m_{x3} = CC(p_o^*(\frac{(e-e_4)^2}{2} + T^{-2}(\ln(1+eT)-\ln(1+e_4T))$$
$$-T^{-1}(1+e_4T)^{-1}(e-e_4)) -\alpha n_o/2(e-e_4)^2-Kn_o/6$$
$$(e^3-3\beta e^2+3e\ e_4^2-6\beta ee_4+2e_4^3-3\beta e_4^2)) + m_o \qquad (4.6.30)$$

From boundary condition (4.6.5)

$$p_o^* = \frac{3\alpha n_o(1-e_4)^2+Kn_o(1-3\beta+6\beta e_4-3e_4^2-3\beta e_4^2 + 2e_4^3)}{3((1-e_4)^2+2T^{-2}(\ln(1+T)-\ln(1+e_4T))-2T^{-1}(1+e_4T)^{-1}(1-e_4))} \qquad (4.6.31)$$

From the continuity conditions (4.6.4)

$$p_o^* = \frac{2\alpha n_o(e_5-e_4)+Kn_o(e_5^2-2e_5\beta-e_4^2+2\beta e_4)}{(e_5+T^{-1}((1+e_5T)^{-1}-(1+e_4T)^{-1}) - e_4)} \qquad (4.6.32)$$

$$CC = \frac{6(m_o+\bar{m}_o)}{3\alpha n_o(e_5-e_4)^2+Kn_o(e_5^3-3\beta e_5^2+3e_5e_4^2-6\beta e_5e_4+2e_4^3-3\beta e_4^2) -3p_o^*((e_5-e_4)^2+2T^{-2}(\ln(1+e_5T)-\ln(1+e_4T))-T^{-1}(1+e_4T)^{-1}(e_5-e_4))} \qquad (4.6.33)$$

4.6.6 Special Case

Let $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

For $e_4 \leq \beta$, equation (4.6.19) will be

$$p_o^* = \frac{3\alpha(1-e_4)^2 + K(1-\beta)^3}{3((1-e_4)^2+2T^{-2}(\ln(1+T)-\ln(1+e_4T))-T^{-1}(1+e_4T)^{-1}(1-e_4))} \tag{4.6.34}$$

when $e_5 \leq \beta$, equations (4.6.20) and (4.6.21) will be

$$p_o^* = \frac{\alpha(e_5-e_4)}{(e_5-e_4)+T^{-1}((1+e_5T)^{-1}-(1+e_4T)^{-1})} \tag{4.6.35}$$

and

$$CC = \frac{4}{\alpha(e_5-e_4)^2-p_o^*((e_5-e_4)^2+2T^{-2}(\ln(1+e_5T)-\ln(1+e_4T)) -2T^{-1}(1+e_4T)^{-1}(e_5-e_4))} \tag{4.6.36}$$

when $e_5 \geq \beta$, equations (4.6.22) and (4.6.23) will be

$$p_o^* = \frac{2\alpha(e_5-e_4) + K(e_5-\beta)^2}{2(e_5-e_4) + 2T^{-1}((1+e_5T)^{-1}-(1+e_4T)^{-1})} \tag{4.6.37}$$

and

$$CC = \frac{12}{(3\alpha(e_5-e_4)^2+K(e_5-\beta)^3-p_o^*(3(e_5-e_4)^2+6T^{-2}(\ln(1+e_5T) -\ln(1+e_4T))-T^{-1}(1+e_4T)^{-1}(e_5-e_4)))} \tag{4.6.38}$$

when $e_4 \geq \beta$, the equations (4.6.31), (4.6.32) and (4.6.33), will be

$$p_o^* = \frac{3\alpha(1-e_4)^2 + K(1-3\beta+6\beta e_4-3e_4^2-3\beta e_4^2+ 2e_4^3)}{3((1-e_4)^2+2T^{-2}(\ln(1+T)-\ln(1+e_4T))-2T^{-1}(1+e_4T)^{-1}(1-e_4))} \tag{4.6.39}$$

$$p_o^* = \frac{2\alpha(e_5-e_4) + K\ (e_5^2 - 2\beta e_5 - e_4^2 + 2\beta e_4)}{(e_5-e_4) + T^{-1}((1+e_5T)^{-1}-(1+e_4T)^{-1})} \qquad (4.6.40)$$

and

$$CC = \frac{12}{3\alpha(e_5-e_4)^2+K(e_5^3-3\beta e_5^2+3e_5e_4^2-6\beta e_5e_4+2e_4^3-3\beta e_4^2)}$$
$$-p_o^*((e_5-e_4)^2+2T^{-2}(\ln(1+e_5T)-\ln(1+e_4T))$$
$$-T^{-1}(1+e_4T)^{-1}(e_5-e_4)) \qquad (4.6.41)$$

when $\alpha = \beta = 1$

$$p_o^* = \frac{(1-e_4)^2}{(1-e_4)^2+2T^{-2}(\ln(1+T)-\ln(1+e_4T))-2T^{-1}(1+e_4T)^{-1}(1-e_4)} \qquad (4.6.38)$$

$$p_o^* = \frac{2(e_5-e_4)}{(e_5-e_4) + T^{-1}((1+e_5T)^{-1}-(1+e_4T)^{-1})} \qquad (4.6.42)$$

and

$$CC = \frac{4}{3(e_5-e_4)^2-p_o^*((e_5-e_4)^2+2T^{-2}(\ln(1+e_5T)-\ln(1+e_4T))}$$
$$-T^{-1}(1+e_4T)^{-1}(e_5-e_4)) \qquad (4.6.43)$$

4.7 CONCLUSION

Three graphs have been drawn between CC and p_o^* for different values of T, α and β. The Fig. 4.7.1 has been drawn for $\alpha = \beta = 1$ and for different T values. It is clear from this graph that as the value of T increases the

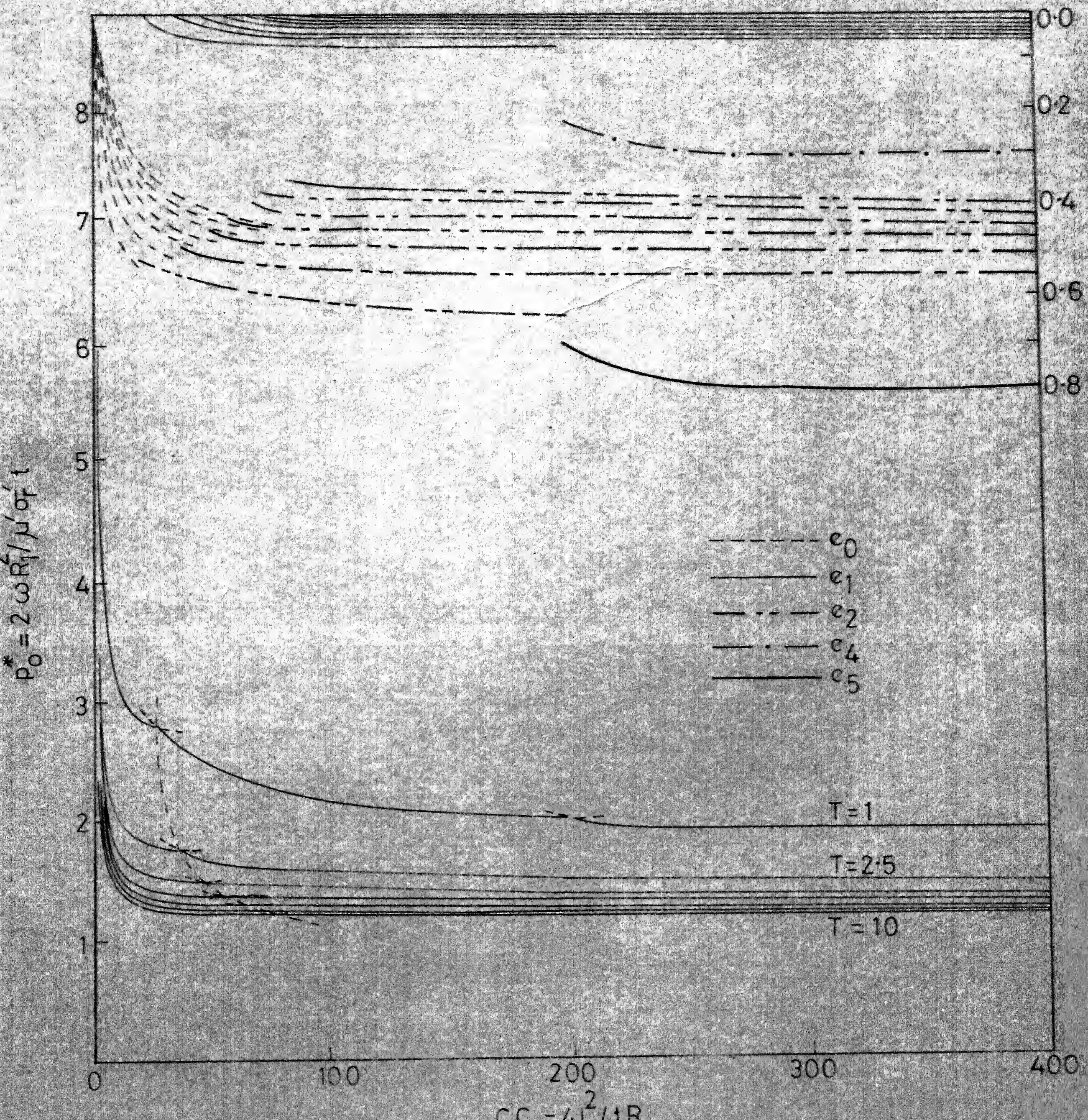


Fig. 4·7·1 p_o^* Vs CC for $\alpha = \beta = 1$ and different T in silos for Reimbert's pressure distribution theory

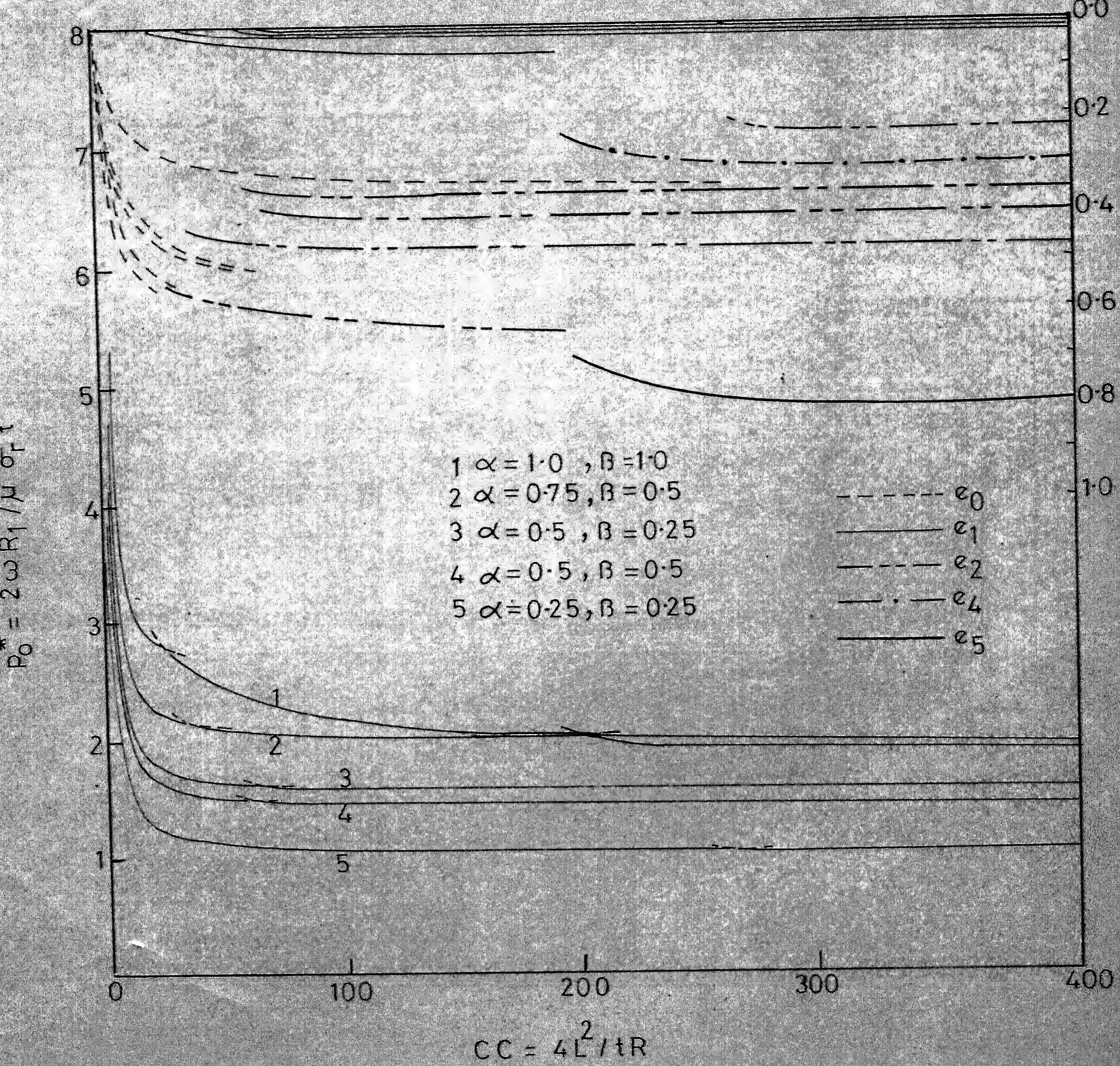


Fig. 4·7·2 p_o^* Vs CC for variable α and β at T=1 in silos for Reimbert's pressure distribution theory

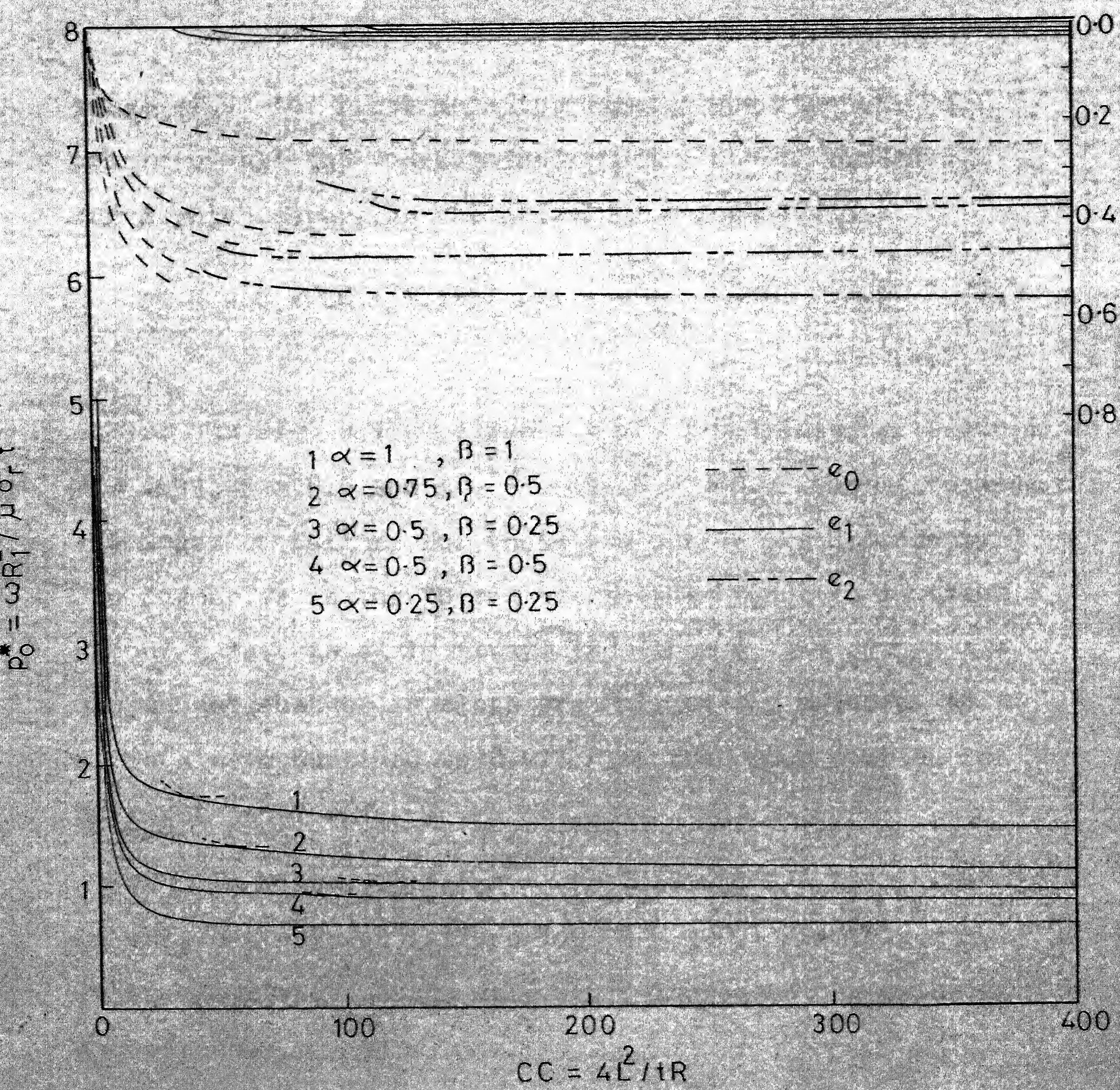


Fig. 4·7·3 p_o^* Vs CC for variable α and β at T=2·5 in silos for Reimbert's pressure distribution theory

range of CC for first mode increases. When the critical points of all the T are joined for $\alpha = \beta = 1$, one gets a curve which separates the mode I from mode II is

$$p_o^* = -6.2098 + 0.1237\ CCR + \frac{158.2258}{CCR} - 0.0007\ CCR^2 \tag{4.7.1}$$

In Fig. 4.7.2, curve $\alpha = \beta = T = 1$ intersects curve $\alpha = 0.75$, $\beta = 0.5$ and $T = 1$, at CC = 208. This is because the modes of failure for these two curves are different. The former is in the third mode whether later is in the second mode. As e_2 is always less then β, the curves are smooth and changes in slope are only at the critical point e_1 is always confined to 0.075 of the upper portion for CC $\leq$ 400 and $T = 1$.

In Fig.4.7.3 for $\alpha = \beta = 0.25$ and $T = 2.5$ there is only one mode of failure for CC < 400. From Fig. 4.7.1 it is clear that as the value of T increases the value of e_2 decreases.

5. ANALYSIS OF REINFORCED CONCRETE SILOS BASED ON THE JANSSEN'S PRESSURE DISTRIBUTION THEORY

5.1 PRESSURE DISTRIBUTION

The material filled in cylindrical silos, in which emptying hole situated at the centre of the bottom, is held in equilibrium by the reaction from the sloping bottom and the frictional force on the vertical wall, as shown in Fig. 5.1.1[13]. Equations are given below.

The vertical static unit pressure at depth X below the surface of the stored material is

$$q = \frac{\gamma R_1}{\mu' k} \left(1 - \exp(-\mu' k\, X/R_1) \right) \tag{5.1.1}$$

The lateral static unit pressure at depth X is

$$p = \frac{\gamma R_1}{\mu'} \left(1 - \exp(-\mu' kX/R_1) \right) \tag{5.1.2}$$

Vertical frictional force per unit width of wall perimeter on wall above depth X

$$V = (\gamma X - 0.8\, q)\, R_1 \tag{5.1.3}$$

where

μ' = $\tan \emptyset'$, coefficient of friction between stored material and silo walls ,and

$$k = \frac{1 - \sin \phi}{1 + \sin \phi},$$

where

ϕ' = angle of friction between material stored and silo wall

ϕ = angle of internal friction of stored material

R_1 = hydraulic radius

γ = unit weight of stored material

Static unit pressure normal to a surface inclined at an angle θ to the horizontal at depth X below surface of stored material is[12]

$$q_\alpha = p \sin^2\theta + q \cos^2\theta \qquad (5.1.4)$$

5.2 EQUILIBRIUM EQUATIONS

The equilibrium equations for a silo in non-dimensional form are given by the equations (2.3.5) and (2.3.6). Unlike water tanks there is a vertical pressure along the walls besides the normal horizontal. The equilibrium equation for the normal pressure is given by

$$\frac{tR}{4L^2} \frac{d^2 m_x}{de^2} + n_\theta = p^* \qquad (5.2.1)$$

where

$$p^* = \frac{2\gamma R_1^2}{\mu' \sigma_x' t} (1-\exp(-kL\mu' X/(R_1 L)))$$

$$= p_o^* (1-\exp(-Te)) \qquad (5.2.2)$$

where

$$p_o^* = \frac{2\gamma\, R_1^2}{\mu'\sigma_r' t} \tag{5.2.3}$$

and

$$T = \frac{\mu' k\, L}{R_1} \tag{5.2.4}$$

Substituting these values in equation (5.2.1) , one gets

$$\frac{d^2 m_x}{de^2} + CC\, n_\theta = CC\, n_\theta = CCp_o^*(1-\exp(-Te)) \tag{5.2.5}$$

5.3 SHORT SILO

5.3.1 Stress Distribution

A shallow silo is also known as short silo. The failure of the silo at collapse is known as more one failure as in tank. The distribution of the stress resultants and the collapse mechanism are shown in Fig. 5.3.1. The distribution of n_θ is as shown in Fig. 5.3.1(c). This is given by the equations

$$\begin{aligned} &\text{Zone I} : && n_\theta = \alpha n_o \text{ for } 0 \leq e \leq \beta \\ &\text{Zone II} : && n_\theta = \alpha n_o + Kn_o(e-\beta) \text{ for } \beta \leq e \leq 1 \end{aligned} \tag{5.3.1}$$

where

$$K = \frac{1-\alpha}{1-\beta}$$

The boundary and the continuity conditions are

$$m_{x1} = 0 , \quad \frac{dm_{x1}}{de} = 0 \text{ at } e = 0 \tag{5.3.2}$$

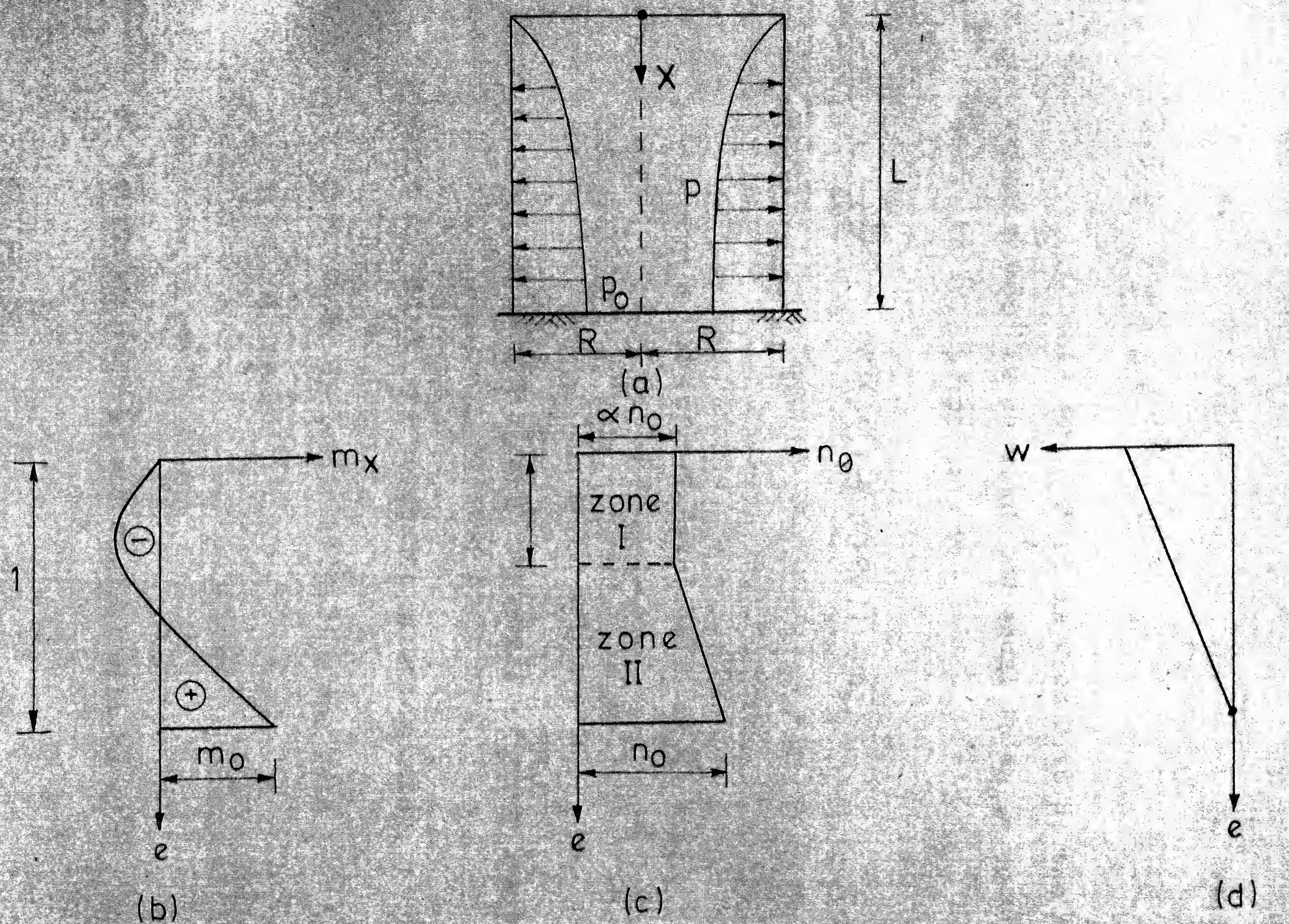


Fig. 5·3·1 Stress distribution and collapse mechanism for short silo

$$m_{x1} = m_{x2} \,,\quad \frac{dm_{x1}}{de} = \frac{dm_{x2}}{de} \quad \text{at } e = \beta \tag{5.3.3}$$

and

$$m_{x1} = m_o \quad \text{at} \quad e = 1 \tag{5.3.4}$$

5.3.2 $0 \leq e \leq \beta$ (Zone I)

In this zone, which is from 0 to β, the circumferential reinforcement is constant and equal to αn_o. So the equilibrium equation (5.2.5) reduces to

$$\frac{d^2 m_{x1}}{de^2} = CC\ (p_o^*(1-\exp(-Te))\ -\alpha n_o) \tag{5.3.5}$$

Integrating once

$$\frac{dm_{x1}}{de} = CC\ (p_o^*(e+T^{-1}\exp(-Te))\ -\alpha n_o e) + A_1 \tag{5.3.6}$$

Integrating once again

$$m_{x1} = CC(p_o^*(e^2/2-T^{-2}\exp(-Te))\ - \frac{\alpha n_o e^2}{2}\ )+A_1 e+B_1 \tag{5.3.7}$$

where A_1 and B_1 are constants of integration.

Using boundary condition (5.3.2) , one gets

$$A_1 = -\ CC\ p_o^*\ T^{-1} \tag{5.3.8}$$

and $B_1 = CC\ p_o^* T^{-2}$ (5.3.9)

So equations (5.3.6) and (5.3.7) will be

$$\frac{dm_{x1}}{de} = CC\ (p_o^*(e+T^{-1}\exp(-Te)\ -T^{-1})-\alpha n_o e) \qquad (5.3.10)$$

and

$$m_{x1} = CC\ (p_o^*(e^2/2-T^{-2}\exp(-Te)-T^{-1}e+T^{-2})-\alpha n_o e^2/2) \qquad (5.3.11)$$

5.3.3 $\beta \leq e \leq 1$ (Zone II)

In this zone, which is from β to 1, the circumferential reinforcement is varying linearly from αn_o to n_o. So the equilibrium equation (5.2.5) reduces to

$$\frac{d^2m_{x2}}{de^2} = CC(p_o^*(1-\exp(-Te))-\alpha n_o-Kn_o(e-\beta)) \qquad (5.3.12)$$

First integration gives

$$\frac{dm_{x2}}{de} = CC(p_o^*(e+T^{-1}\exp(-Te))-\alpha n_o e-Kn_o(e^2/2-\beta e)) + A_2 \qquad (5.3.13)$$

Second integration gives

$$m_{x2} = CC\ (p_o^*(e^2/2-T^{-2}\exp(-Te))-\alpha n_o e^2/2-Kn_o(e^3/6-\beta e^2/2)) + A_2 e + B_2 \qquad (5.3.14)$$

where A_2 and B_2 are constants of integration.

The continuity conditions (5.3.3) gives

$$A_2 = -\ CC\ (T^{-1}p_o^* + Kn_o\beta^2/2) \qquad (5.3.15)$$

and $B_2 = CC\ (T^{-2}p_o^* + Kn_o\beta^3/6)$ (5.3.16)

So equations (5.3.13) and (5.3.14) will be

$$\frac{dm_{x2}}{de} = CC(p_o^*(e+T^{-1}\exp(-Te)-T^{-1})-\alpha n_o e-Kn_o/2(e-\beta)^2) \tag{5.3.17}$$

and

$$m_{x2} = CC\ (p_o^*(e^2/2-T^{-2}\exp(-Te)-T^{-1}e+T^{-2})-\alpha n_o e^2/2-Kn_o(e-\beta)^3/6) \tag{5.3.18}$$

Hence from boundary condition (5.3.4)

$$p_o^* = \frac{6m_o/CC + 3\alpha n_o+Kn_o(1-\beta)^3}{3(1+2T^{-2}(1-\exp(-T))-2T^{-1})} \tag{5.3.19}$$

By using the equations (5.3.11) and (5.3.18), the bending moment distribution will be as shown in Fig. 5.3.1(b).

5.3.4 Critical Condition

As the L/R ratio of the shell increases, the value of CC as well as T increases. If L/R ratio remains same, but L and R increase , the value of CC increases but T remains constant. As CC increases, p_o^* decreases, so m_{x1} and m_{x2} will also decrease and it will reach the maximum negative value for certain CC = CCR and certain $e = e_o$.

So for $e = e_o$

$$m_x = -\bar{m}_o \text{ and } dm_x/de = 0 \tag{5.3.20}$$

when $e_o \leq \beta$

$$p_o^* = \alpha n_o e_o/(e_o+T^{-1}\exp(-Te_o)-T^{-1}) \tag{5.3.21}$$

and

$$CCR = \frac{2\bar{m}_o}{\alpha n_o e_o^2 - p_o^*(e_o^2+2T^{-2}(1-\exp(-Te_o))-2T^{-1}e_o)} \tag{5.3.22}$$

when $\beta \leq e_o \leq 1$

$$p_o^* = \frac{2\alpha n_o e_o + Kn_o(e_o-\beta)^2}{2(e_o+T^{-1}\exp(-Te_o)-T^{-1})} \tag{5.3.23}$$

and

$$CCR = \frac{6\bar{m}_o}{3\alpha n_o e_o^2+Kn_o(e_o-\beta)^3-p_o^*(3e_o^2+6T^{-2}(1-\exp(-Te_o))-6T^{-1}e_o)} \tag{5.3.24}$$

5.3.5 Special Case

Let $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

Then

$$p_o^* = \frac{6/CC+3\alpha+K(1-\beta)^3}{3(1+2T^{-2}(1-\exp(-T)-2T^{-1})} \tag{5.3.25}$$

when $e_o \leq \beta$

$$p_o^* = \alpha e_o/(e_o+T^{-1}\exp(-Te_o)-T^{-1}) \tag{5.3.26}$$

$$CCR = \frac{2}{\alpha e_o^2-p_o^*(e_o^2+2T^{-2}(1-\exp(-Te_o))-2T^{-1}e_o)} \tag{5.3.27}$$

when $\beta \leq e_o \leq 1$

$$p_o^* = \frac{2\alpha e_o + K(e_o-\beta)^2}{2(e_o+T^{-1}\exp(-Te_o)-T^{-1})} \tag{5.3.28}$$

and

$$CCR = \frac{6}{3\alpha e_o^2+K(e_o-\beta)^3-p_o^*(3e_o^2+6T^{-2}(1-\exp(-Te_o))-6T^{-1}e_o)} \tag{5.3.29}$$

when $\alpha = \beta = 1$

$$p_o^* = \frac{1 + 2/CC}{(1 + 2T^{-2}(1-\exp(-T))-2T^{-1})} \tag{5.3.30}$$

$$p_o^* = \frac{e_o}{(e_o + T^{-1}\exp(-Te_o)-T^{-1})} \tag{5.3.31}$$

and

$$CCR = \frac{2}{(e_o^2-p_o^*(e_o^2+2T^{-2}(1-\exp(-Te_o))-2T^{-1}e_o)} \tag{5.3.32}$$

5.3.6 Critical Values

The distribution of n_θ is dependent upon the values of α and β. For a given set of α and β values and for pre-cribed T, the critical shell parameter value CCR, the location of hinge e_o and collapse pressure p_o^* are shown in Tables 5.1, 5.2 and 5.3 for $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$ and for T equal to 1, 2.5 and 4 respectively.

TABLE 5.1 : CCR FOR FIRST MODE OF FAILURE OF SILOS AT $T = 1$ AND $m_o = \bar{m}_o = n_o = 1$

α	β	CCR	e_o	p_o^*
0.25	0.25	169.00	0.4365	1.5232
0.50	0.25	49.20	0.5497	2.4014
0.50	0.50	45.80	0.5358	2.2178
0.75	0.50	27.75	0.5628	3.1965
1.00	1.00	19.90	0.5719	4.1628

TABLE 5.2 : CCR FOR FIRST MODE OF FAILURE OF SILOS AT $T = 2.5$ AND $m_o = \bar{m}_o = n_o = 1$

α	β	CCR	e_o	p_o^*
0.25	0.25	266.0	0.335	0.8064
0.50	0.25	67.5	0.498	1.2628
0.50	0.50	59.2	0.499	1.1653
1.00	1.00	24.95	0.5479	2.1874

TABLE 5.3 : OCR FOR FIRST MODE OF FAILURE OF SILOS AT $T = 4$ and $m_o = \bar{m}_o = n_o = 1$

α	β	CCR	e_o	p_o^*
0.25	0.25	382.0	0.276	0.6357
0.50	0.25	90.2	0.447	0.9887
0.50	0.50	75.0	0.468	0.9125
0.75	0.50	43.5	0.507	1.3125
1.00	1.00	30.5	0.5259	1.7129

From these tables it is clear that as the value of T increases the range of CCR for first mode increases. T increases means L/R increases so the rigidity increases.

5.4 MEDIUM SILO

5.4.1 Stress Distribution

Failure in this silo will be of mode two type as in medium tank. The stress distribution and collapse mechanism are as shown in Fig. 5.4.1. The distribution of n_θ is as shown in Fig. 5.4.1(b). There are three zones:

Zone I : $n_o = -\alpha\bar{n}_o$ for $0 \leq e \leq e_1$

Zone II : $n_\theta = \alpha n_o$ for $e_1 \leq e \leq \beta$

Zone III : $n_\theta = \alpha n_o + K(e-\beta)\, n_o$ for $\beta \leq e \leq 1$ (5.4.1)

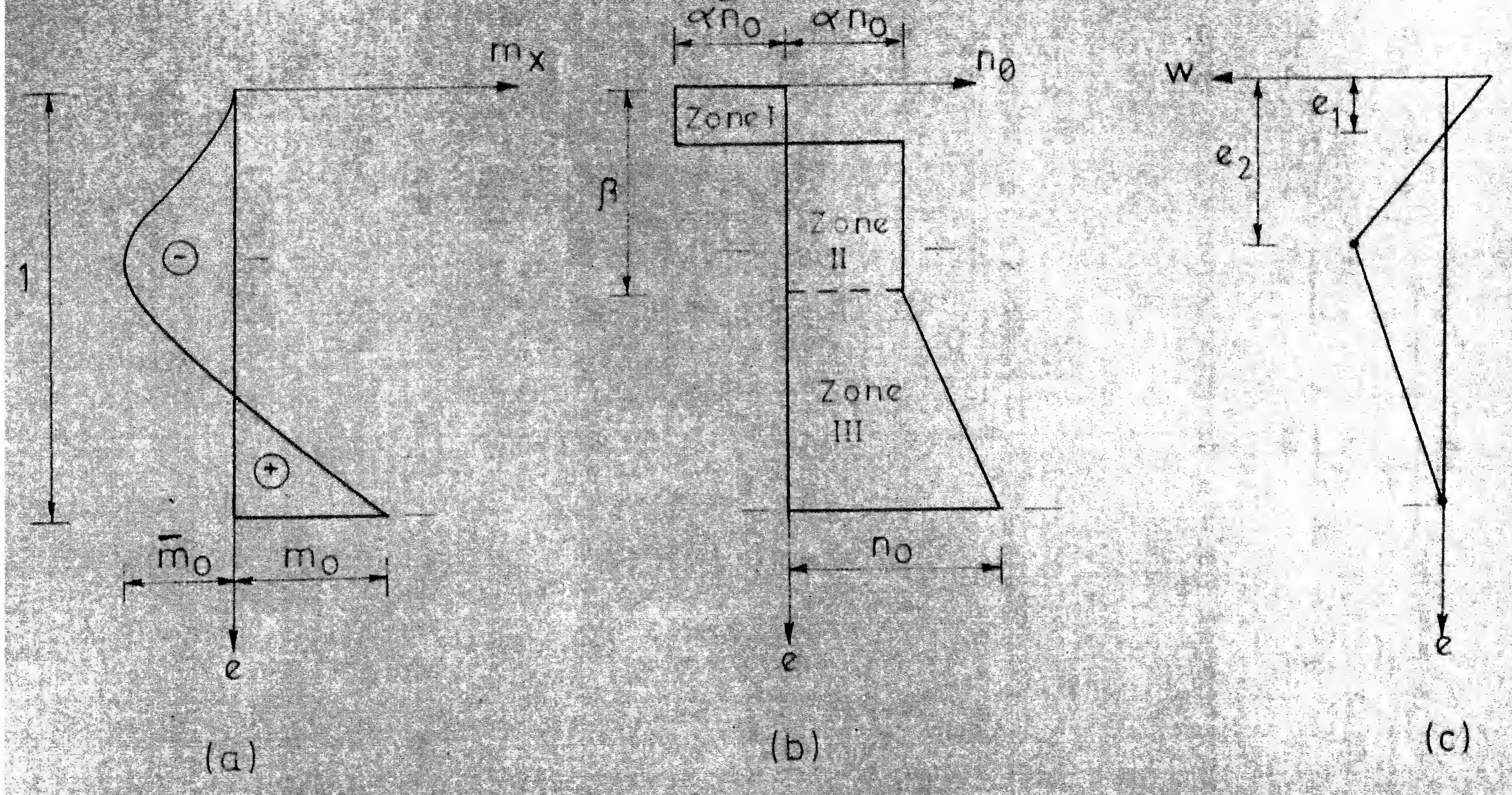


Fig. 5·4·1 Stress distribution and collapse mechanism for medium silo

The boundary and the continuity conditions are

$$m_{x1} = 0 \,, \quad \frac{dm_{x1}}{de} = 0 \text{ at } e = 0 \tag{5.4.2}$$

$$m_{x1} = m_{x2} \,; \quad \frac{dm_{x1}}{de} = \frac{dm_{x2}}{de} \text{ at } e = e_1 \tag{5.4.3}$$

$$m_{x2} = m_{x3} \,, \quad \frac{dm_{x2}}{de} = \frac{dm_{x3}}{de} \text{ at } e = \beta \tag{5.4.4}$$

The continuity condition at $e = e_2$ depends on whether $e_2 \gtrless \beta$.

If $e_2 \leq \beta$

$$m_{x2} = -\bar{m}_o \text{ and } \frac{dm_{x2}}{de} = 0 \quad \text{at} \quad e = e_2$$

and for $e_2 \geq \beta$

$$m_{x3} = -\bar{m}_o \text{ and } \frac{dm_{x3}}{de} = 0 \text{ at } e = e_2 \tag{5.4.5}$$

and

$$m_{x3} = m_o \quad \text{at} \quad e = 1 \tag{5.4.6}$$

5.4.2 $0 \leq e \leq e_1$ (Zone I)

In this zone, which is from 0 to e_1, the circumferential stress is negative and reinforcement provided is constant and equal to $\alpha\bar{n}_o$. So the equilibrium equation (5.2.5) will be

$$\frac{d^2 m_{x1}}{de^2} = CC\ (p_o^*(1-\exp(-Te)) + \alpha\bar{n}_o) \tag{5.4.7}$$

Integrating once

$$\frac{dm_{x1}}{de} = CC\ (p_o^*(e+T^{-1}\exp(-Te)) + \alpha\bar{n}_o e) + A_1 \qquad (5.4.8)$$

Second integration gives

$$m_{x1} = CC\ (p_o^*(e^2/2-T^{-2}\ \exp(-Te)) + \alpha\bar{n}_o e^2/2) + A_1 e + \beta_1 \qquad (5.4.9)$$

where A_1 and B_1 are constants of integration.

From the boundary conditions (5.4.3), one gets

$$A_1 = -\ CC\ p_o^*\ T^{-1} \qquad (5.4.10)$$

$$B_1 = CC\ p_o^* T^{-2} \qquad (5.4.11)$$

So equation (5.4.8) and (5.4.9) reduce to

$$\frac{dm_{x1}}{de} = CC(p_o^*(e+T^{-1}\ \exp(-Te)\ -T^{-1}) + \alpha\bar{n}_o e) \qquad (5.4.12)$$

and

$$m_{x1} = CC(p_o^*(e^2/2+T^{-2}(1-\exp(-Te))-T^{-1}e)+ \alpha\bar{n}_o e^2/2) \qquad (5.4.13)$$

5.4.3 $e_1 \leq e \leq \beta$ (Zone II)

In this zone, which is from e_1 to β, the circumferential stress is positive and reinforcement provided is constant equal to αn_o from e_1 to β. So the equilibrium equation (5.2.5) will be

$$\frac{d^2m_{x2}}{de^2} = CC\ (p_o^*(1-\exp(-Te)) - \alpha n_o) \qquad (5.4.14)$$

First integration gives

$$\frac{dm_{x2}}{de} = CC(p_o^*(e+T^{-1}\exp(-Te))-\alpha n_o e) + A_2 \qquad (5.4.15)$$

Integrating once again

$$m_{x2} = CC(p_o^* (e^2/_2-T^{-2}\exp(-Te))- \alpha n_o e^2/2) + A_2 e+B_2 \qquad (5.4.16)$$

where A_2 and B_2 are constants of integration.

Using the continuity conditions (5.4.3), one gets

$$A_2 = CC (-p_o^* T^{-1}+\alpha e_1 (n_o+\bar{n}_o)) \qquad (5.4.17)$$

and

$$B_2 = CC(p_o^* T^{-2}-\alpha e_1^2/2 (n_o+\bar{n}_o)) \qquad (5.4.18)$$

Substituting these values in equations (5.4.15) and (5.4.16)

$$\frac{dm_{x2}}{de} = CC (p_o^*(e+T^{-1}\exp(-Te) -T^{-1})-\alpha n_o e+\alpha e_1 (n_o+\bar{n}_o)) \qquad (5.4.19)$$

and

$$m_{x2} = CC(p_o^*(e^2/2-T^{-2}\exp(-Te) -T^{-1}e + T^{-2}) -\alpha n_o e^2/2+(\alpha e e_1-\alpha e_1^2/2) (n_o+\bar{n}_o)) \qquad (5.4.20)$$

5.4.4 $\beta \leq e \leq 1$ (Zone III)

In this zone, which is from β to 1, the circumferential stress is positive and reinforcement provided

is linearly varying from αn_o to n_o from β to 1. So the equilibrium equation (5.3.5) will be

$$\frac{d^2 m_{x3}}{de^2} = CC(p_o^*(1-\exp(-Te)) - \alpha n_o - Kn_o(e-\beta)) \quad (5.4.21)$$

First integration gives

$$\frac{dm_{x3}}{de} = CC\ (p_o^*(e+T^{-1}\exp(-Te)) - \alpha n_o e - Kn_o(e^2/2-\beta e)) + A_3 \quad (5.4.22)$$

Second integration gives

$$m_{x3} = CC\ (p_o^*\ (e^2/2-T^{-2}\ \exp(-Te)) - \alpha n_o e^2/2 - Kn_o(e^3/6-\beta e^2/2)) + A_3 e + B_3 \quad (5.4.23)$$

where A_3 and B_3 are constants of integration.

Using the continuity conditions (5.4.4).

$$A_3 = -CC(T^{-1}\ p_o^* + Kn_o\beta^2/2 + \alpha e_1(n_o+\bar{n}_o)) \quad (5.4.24)$$

and

$$B_3 = CC\ (p_o^* T^{-2} + Kn_o\beta^3/6 - \alpha e_1^2/2(n_o+\bar{n}_o)) \quad (5.4.25)$$

Substituting these values in equations (5.4.22) and (5.4.23).

$$\frac{dm_{x3}}{de} = CC(p_o^*\ (e+T^{-1}\exp(-Te)-T^{-1}) - \alpha n_o e - Kn_o/2(e-\beta)^2 + \alpha e_1(n_o+\bar{n}_o)) \quad (5.4.26)$$

$$m_{x3} = CC(p_o^*(e^2/2-T^{-2}\exp(-Te)-T^{-1}e+T^{-2})-\alpha n_o e^2/2$$

$$Kn_o/6(e-\beta)^3+(n_o+\bar{n}_o)\alpha(e_1 e-e_1^2/2)) \qquad (5.4.27)$$

Using the boundary condition (5.4.6) in the equation (5.4.27)

$$p_o^* = \frac{6m_o/CC+3\alpha n_o+Kn_o(1-\beta)^3-3\alpha(n_o+\bar{n}_o)(2e_1-e_1^2)}{3(1-2T^{-2}\exp(-T)-2T^{-1}+2T^{-2})} \qquad (5.4.28)$$

From the first condition of (5.4.5) , one gets

$$p_o^* = \frac{\alpha n_o e_2 - \alpha e_1(n_o+\bar{n}_o)}{(e_2+T^{-1}\exp(-Te_2)-T^{-1})} \qquad (5.4.29)$$

and

$$CC = \frac{2\bar{m}_o}{(\alpha n_o e_2^2-(n_o+\bar{n}_o)\alpha(2e_1e_2-e_1^2)-p_o^*(e_2^2-2T^{-2}\exp(-Te_2)-2T^{-1}e_2+2T^{-2})} \qquad (5.4.30)$$

From second condition of (5.4.5)

$$p_o^* = \frac{2\alpha n_o e_2+Kn_o(e_2-\beta)^2-\alpha e_1(n_o+\bar{n}_o)}{2(e_2+T^{-1}\exp(-Te_2)-T^{-1})} \qquad (5.4.31)$$

and

$$CC = \frac{6\bar{m}_o}{(3\alpha n_o e_2^2+Kn_o(e_2-\beta)^3-3\alpha(n_o+\bar{n}_o)(2e_1e_2-e_1^2)-p_o^*(3e_2^2-6T^{-2}\exp(-Te_2)-6T^{-1}e_2+6T^{-2}))} \qquad (5.4.32)$$

By using equations (5.4.13), (5.4.20) and (5.4.23), the bending moment distribution will be as shown in Fig.5.4.1(a)

5.4.5 Critical Condition

As the value of CC increases the positive moment above the negative hinge will increase and it will reach the positive maximum moment m_o for certain value of CC equal to CCR at $e = e_3$. The value of CCR will also depend on whether $e_3 \gtrless \beta$. e_1 is confined to within 0.1 L of the upper portion so there is no point in considering $\beta < e_1$.

So at $e = e_3$

$$m_x = m_o \text{ and } \frac{dm_x}{de} = 0 \tag{5.4.33}$$

when

$e_3 \leq \beta$

$$p_o^* = \frac{\alpha n_o(e_2-e_3)}{(e_2-e_3) + T^{-1}(\exp(-Te_2) - \exp(-Te_3))} \tag{5.4.34}$$

and

$$CCR = \frac{2m_o}{(p_o^*(e_3^2+2T^{-2}(1-\exp(-Te_3))-2T^{-1}e_3) - \alpha n_o e_3^2 + \alpha(n_o+\bar{n}_o)(2e_1e_3-e_1^2))} \tag{5.4.35}$$

when $\beta \leq e_3 \leq e_2$

$$p_o^* = \frac{(2\alpha n_o+Kn_o)(e_2+e_3)-2Kn_o\beta)(e_2-e_3)}{2((e_2-e_3)+T^{-1}(\exp(-Te_2)-\exp(-Te_3))} \tag{5.4.36}$$

and $CCR \doteq 6m_o/(p_o^*(3e_3^2+6T^{-2}(1-\exp(-Te_3))-6T^{-1}e_3)-3\alpha n_o e_3^2$
$$-Kn_o(e_3-\beta)^3+3\alpha(n_o+\bar{n}_o)(2e_1e_3-e_1^2)) \tag{5.4.37}$$

5.4.6 Special Case

Let $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

Then equation (5.4.28) will be

$$p_o^* = \frac{6/CC + 3\alpha + K(1-\beta)^3 - 6\alpha e_1 \ (2 - e_1 \ )}{(3+6T^{-2}(1-\exp(-T)) -6T^{-1}} \tag{5.4.38}$$

when $e_2 \leq \beta$, equations (5.4.29) and (5.4.30) will be

$$p_o^* = \frac{\alpha(e_2 - 2e_1)}{(e_2 + T^{-1}(\exp(-Te_2) - 1))} \tag{5.4.39}$$

and

$$CC = \frac{2}{(\alpha e_2^2 - 2\alpha(2e_1e_2 - e_1^2) - p_o^*(e_2^2 - 2T^{-2}\exp(-Te_2) -2T^{-1}e_2 + 2T^{-2}))} \tag{5.4.40}$$

when $e_2 \geq \beta$, equations (5.4.31) and (5.4.32) will be

$$p_o^* = \frac{2\alpha e_2 + K(e_2-\beta)^2 -4\alpha e_1}{2(e_2 + T^{-1}(\exp(-Te_2) -1))} \tag{5.4.41}$$

and

$$CC = \frac{6}{(3\alpha e_2^2 + K(e_2-\beta)^3 - 6\alpha(2e_1e_2 - e_1^2) - p_o^*(3e_2^2 + 6T^{-2}(1-\exp(Te_2)) - 6T^{-1}e_2))} \tag{5.4.42}$$

when $e_3 \leq \beta$, equations (5.4.34) and (5.4.35) reduce to

$$p_o^* = \frac{\alpha(e_2 - e_3)}{((e_2 - e_3) + T^{-1}(\exp(-Te_2) - \exp(-Te_3)))} \tag{5.4.43}$$

and

$$\text{CCR} = \frac{2}{(p_o^*(e_3^2+2T^{-2}(1-\exp(-Te_3)-Te_3))-\alpha e_3^2+2\alpha(2e_1e_3-e_1^2))} \tag{5.4.44}$$

when $\beta \leq e_3$, equations (5.4.36) and (5.4.37) will be

$$p_o^* = \frac{(2\alpha+K(e_2+e_3)-2K\beta)(e_2-e_3)}{2((e_2-e_3)+T^{-1}(\exp(-Te_2)-\exp(-Te_3))} \tag{5.4.45}$$

and

$$\text{CCR} = \frac{6}{(p_o^*(3e_3^2+6T^{-2}(1-\exp(-Te_3)\ \ Te_3))-3\alpha e_3^2-K(e_3-\beta)^3+6\alpha(2e_1e_3-e_1^2))} \tag{5.4.46}$$

when $\alpha = \beta = 1$

$$p_o^* = \frac{2/CC + 1-2e_1\ (2-e_1)}{(1+2T^{-2}(1-\exp(-T))-2T^{-1}\ )} \tag{5.4.47}$$

$$p_o^* = \frac{(e_2-2e_1)}{(e_2+T^{-1}\exp(-Te_2)-T^{-1}\ )} \tag{5.4.48}$$

$$CC = \frac{2}{(e_2^2-2(2e_1e_2-e_1^2)-p_o^*(e_1^2+2T^{-2}(1-\exp(Te_2))-2T^{-1}e_2\ )} \tag{5.4.49}$$

$$p_o^* = \frac{(e_2-e_3)}{((e_2-e_3)+T^{-1}(\exp(-Te_2)-\exp(-Te_3\ ))} \tag{5.4.50}$$

$$\text{CCR} = \frac{2}{(p_o^*(e_3^2+2T^{-2}(1-\exp(-Te_3))-2T^{-1}e_3)-e_3^2+2(2e_1e_3-e_1^2))} \tag{5.4.51}$$

5.4.7 Critical Values

The distribution of n_θ is dependent upon the values of α and β. For a given set of α and β values at certain T the critical shell parameter value CCR, location of hinges e_2 and e_3, the range of hoop compression from 0 to e_1 and the collapse pressure p_o^* are shown in Tables 5.4 and 5.5 for $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$ and T equal to 1 and 2.5 respectively.

TABLE 5.4 : CCR FOR SECOND MODE OF FAILURE OF SILOS AT $T = 1$ AND $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

α	β	CCR	e_1	e2	e_3	p_o^*
0.25	0.25	-	-	-	-	-
0.50	0.25	-	-	-	-	-
0.50	0.50	-	-	-	-	-
0.75	0.5	299.0	0.065	0.595	0.212	2.26
1.00	1.0	119.5	0.075	0.750	0.315	2.70

TABLE 5.5 : CCR FOR SECOND MODE OF FAILURE OF SILOS AT $T=2.5$ AND $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

α	β	CCR	e_1	e_2	e_3	p_o^*
0.25	0.25	-	-	-	-	-
0.50	0.25	-	-	-	-	-
0.25	0.50	-	-	-	-	-
0.75	0.50	92	0.041	0.55	0.202	1.36
1.00	1.00	275	0.060	0.525	0.288	1.62

As T increases, the zone for second mode for $\alpha = 0.75$ and $\beta = 0.5$ decreases.

5.5 DEEP SILO

5.5.1 Stress Distribution

Failure in this mode is known as mode three failure and also known as 'partial failure' or 'partial collapse', same as in the case of tank.

The stress distribution and collapse mechanism are as shown in Fig. 5.5.1.

The distribution of n_θ is as shown in Fig.5.5.1(b). This is given by the equations

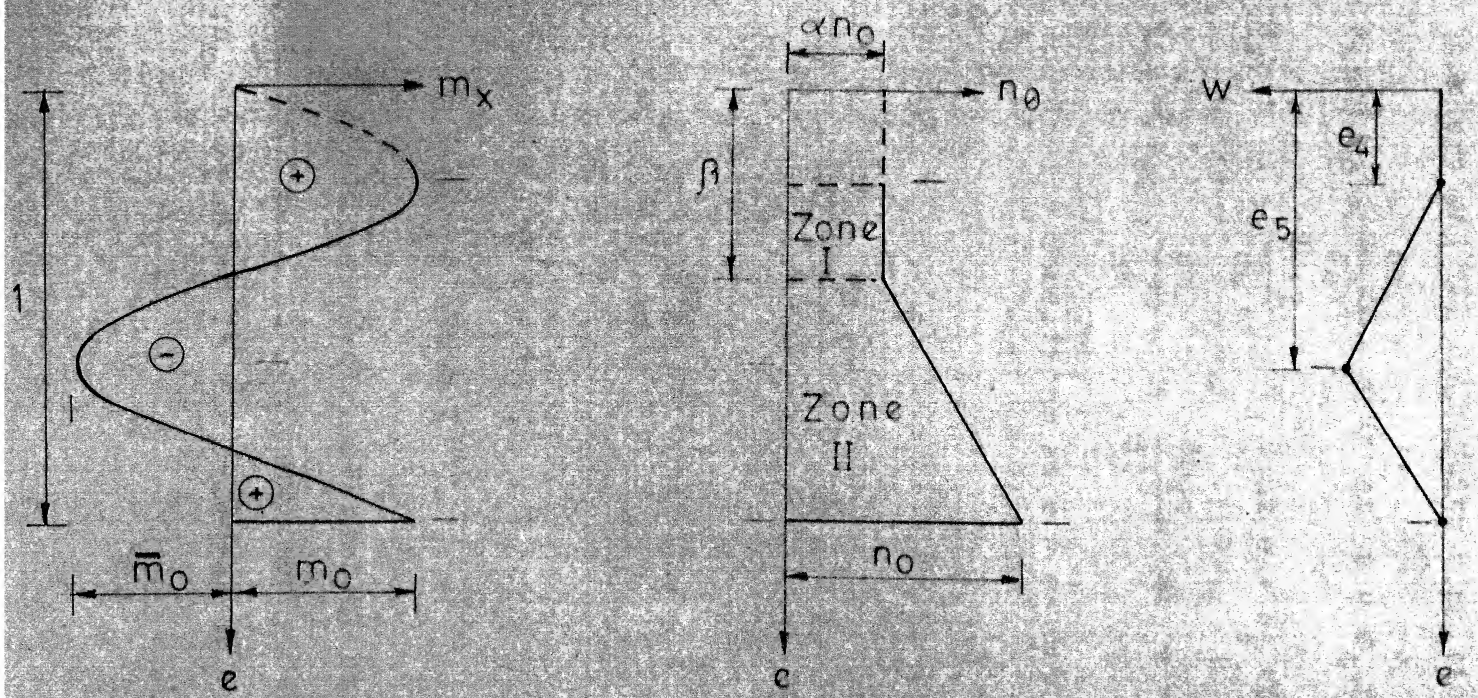


Fig.5·5·1 Stress distribution and collapse mechanism for deep silo

Zone I : $n_\theta = \alpha n_o$ for $e_4 \leq e \leq \beta$

Zone II : $n_\theta = \alpha n_o + K\,(e-\beta)\, n_o$ for $\beta \leq e \leq 1$ (5.5.1)

The boundary and the continuity conditions are

$$m_{x1} = m_o \;,\; \frac{dm_{x1}}{de} = 0 \quad \text{at} \quad e = e_4 \text{ for } \; e_4 \leq \beta$$

$$m_{x2} = m_o \;,\; \frac{dm_{x2}}{de} = 0 \text{ at } e = e_4 \quad \text{for} \quad e_4 \geq \beta$$

$$m_{x1} = m_{x2} \;,\; \frac{dm_{x1}}{de} = \frac{dm_{x2}}{de} \quad \text{at} \quad e = \beta \text{ for } e_4 \leq \beta \qquad (5.5.3)$$

The continuity conditions at $e = e_5$ depends on whether $e_5 \gtrless \beta$

For $e_5 \leq \beta$

$$m_{x1} = -\bar{m}_o \;,\; \frac{dm_{x1}}{de} = 0 \quad \text{at } e = e_5$$

and for $e_5 \geq \beta$

$$m_{x2} = -\bar{m}_o \;,\; \frac{dm_{x2}}{de} = 0 \quad \text{at } e = e_5 \qquad (5.5.4)$$

and

$$m_{x2} = m_o \text{ at } e = 1 \qquad (5.5.5)$$

5.5.2 $e_4 \leq e \leq \beta$ (Zone I)

In this zone, circumferential stress is positive and reinforcement in constant throughout, from e_4 to β, is αn_o so equation (5.2.5) reduces to

$$d^2m_{x1}/de^2 = CC(p_o^*(1-\exp(-Te))-\alpha n_o) \quad (5.5.6)$$

Integrating once

$$dm_{x1}/de = CC(p_o^*(e+T^{-1}\exp(-Te))-\alpha n_o e) + A_1 \quad (5.5.7)$$

Integrating once again

$$m_{x1} = CC\ (p_o^*\ (e^2/2-T^{-2}\ \exp(-Te))-\alpha n_o e^2/2) + A_1 e+B_1 \quad (5.5.8)$$

where A_1 and B_1 are constants of integration.

From the boundary conditions (5.5.2)

$$A_1 = -\ CC(p_o^*(e_4+T^{-1}\ \exp(-Te_4))-\alpha n_o e_4) \quad (5.5.9)$$

$$B_1 = m_o + CC(p_o^*(e_4^2/2+e_4 T^{-1}\exp(-Te_4))+ T^{-2}\exp(-Te_4))-\alpha n_o e_4^2/2) \quad (5.5.10)$$

Substituting these values in equations (5.5.7) and (5.5.8), one gets

$$\frac{dm_{x1}}{de} = CC(p_o^*(e+T^{-1}(\exp(-Te)\ -\exp(Te_4))-\alpha n_o(e-e_4)) \quad (5.5.11)$$

and

$$m_{x1} = CC(p_o^*(\frac{(e-e_4)^2}{2} -T^{-1}\exp(-Te_4)(e-e_4)-T^{-2}(\exp(-Te)\ -\exp(-Te_4))) - \frac{\alpha n_o}{2}(e-e_4)^2) + m_o \quad (5.5.12)$$

5.5.3 $\beta \leq e \leq 1$ (Zone II)

In this zone the circumferential stress is positive from β to 1, and the reinforcement provided in this zone is linearly varying from αn_o to n_o. So the equation (5.2.5) will be

$$d^2m_{x2}/\ de^2 = CC(p_o^*(1-\exp(-Te))-\alpha n_o-Kn_o(e-\beta)) \quad (5.5.13)$$

First integration gives

$$dm_{x2}/de = CC(p_o^*(e+T^{-1}\exp(-Te))-\alpha n_o e-Kn_o(e^2/2-\beta e)) + A_2 \quad (5.5.14)$$

Second integration gives

$$m_{x2} = CC(p_o^*(e^2/2-T^{-2}\exp(-Te)) -\alpha n_o e^2/2-Kn_o(e^3/6-\beta e^2/2)) +A_2 e +B_2 \quad (5.5.15)$$

where A_2 and B_2 are constants of integration.

From the continuity conditions (5.5.3)

$$A_2 = CC(p_o^*(-e_4-T^{-1}\exp(-Te_4))+\alpha n_o e_4-Kn_o\beta^2/2) \quad (5.5.16)$$

$$B_2 = CC(p_o^*(e_4^2/2+T^{-1}e_4\ \exp(-Te_4) + T^{-2}\exp(-Te_4)) -\alpha n_o e_4^2/2 + Kn_o\beta^3/6) + m_o \quad (5.5.17)$$

Substituting these values in equations (5.5.14) and (5.5.15) , one gets

$$\frac{dm_{x2}}{de} = CC(p_o^*((e-e_4) + T^{-1}(\exp(-Te) - \exp(-Te_4)))$$
$$-\alpha n_o(e-e_4) - Kn_o/2(e-\beta)^2) \qquad (5.5.18)$$

and

$$m_{x2} = CC\ (p_o^*\ (\frac{(e-e_4)^2}{2} - T^{-2}(\exp(-Te) - \exp(-Te_4))$$
$$-T^{-1}\ \exp(-Te_4)\ (e-e_4)) - \alpha n_o/2(e-e_4)^2$$
$$-Kn_o/6(e-\beta)^3) + m_o \qquad (5.5.19)$$

Using the boundary condition (5.5.5) in equation (5.5.19)

$$p_o^* = \frac{3\alpha n_o(1-e_4)^2 + Kn_o(1-\beta)^3}{(3(1-e_4)^2 - 6T^{-2}(\exp(-T) - \exp(-Te_4)) - 6T^{-1}\exp(-Te_4)(1-e_4))}$$
$$(5.5.20)$$

5.5.4 $e_4 \leq \beta$ (Case I)

For $e_5 \leq \beta$. Using the boundary and the continuity conditions (5.5.2) and (5.5.4) in (5.5.11)

$$p_o^* = \frac{\alpha n_o(e_5-e_4)}{(e_5-e_4) + (\exp(-Te_5) - \exp(-Te_4))\ T^{-1}}$$

$$CC = \frac{2(m_o+\bar{m}_o)}{(\alpha n_o(e_5-e_4)^2 - p_o^*((e_5-e_4)^2 - 2T^{-1}\exp(-Te_4)\ (e_5-e_4) - 2T^{-2}(\exp(-Te_5) - \exp\ (-Te_4))))} \qquad (5.5.22)$$

when $\beta \leq e_5 \leq 1$

Using the boundary and the continuity conditions (5.5.2) and (5.5.4) in equations (5.5.18) and (5.5.19), one gets

$$p_o^* = \frac{2\alpha n_o(e_5-e_4) + Kn_o(e_5-\beta)^2}{2((e_5-e_4) + T^{-1}(\exp(-Te_5) - \exp(-Te_4)))} \qquad (5.5.23)$$

and

$$CC = \frac{6(m_o+\bar{m}_o)}{(3\alpha n_o(e_5-e_4)^2+Kn_o(e_5-\beta)^3-p_o^*(3(e_5-e_4)^2-6T^{-2}(\exp(-Te_5)-\exp(-Te_4))-6T^{-1}\exp(-Te_4)(e_5-e_4)))}$$

(5.5.24)

5.5.5 $\beta \leq e_4$ (Case II)

For this case, constants of integration A_2 and B_2 for equations (5.5.14) and (5.5.15) from boundary conditions (5.5.2) will be

$$A_2 = -CC(p_o^*(e_4+T^{-1}\exp(-Te_4))-\alpha n_o e_4-Kn_o(e_4^2/2-\beta e_4)) \qquad (5.5.28)$$

and

$$B_2 = m_o+CC(p_o^*(e_4^2/2+T^{-2}\exp(-Te_4) + T^{-1}e_4\exp(-Te_4)) -\alpha n_o e_4^2/2-Kn_o(e_4^3/3-\beta e_4^2/2)) \qquad (5.5.29)$$

Substituting these values in (5.5.14) and (5.5.15), one gets

$$\frac{dm_{x2}}{de} = CC(p_o^*(e+T^{-1}\exp(-Te)-e_4-T^{-1}\exp(-Te_4)) -\alpha n_o(e-e_4) -Kn_o(e^2/2-\beta e-e_4^2/2+\beta e_4)) \qquad (5.5.30)$$

$$m_{x2}=CC(p_o^*(\frac{(e-e_4)^2}{2}-T^{-2}(\exp(-Te)-\exp(-Te_4))-T^{-1}\exp(-Te_4)(e-e_4))$$

$$-\alpha n_o/2(e-e_4)^2-Kn_o/6(e^3-3\beta e^2+3ee_4^2-6\beta ee_4+2e_4^3-3\beta e_4^2)+m_o$$

(5.5.31)

Using the continuity conditions (5.5.5)

$$p_o^* = \frac{3\alpha n_o(1-e_4)^2+Kn_o(1-3\beta+6\beta e_4-3\beta e_4^2-3e_4^2+2e_4^3)}{(3(1-e_4)^2-6T^{-2}(\exp(-T)-\exp(-Te_4))-6T^{-1}\exp(-Te_4)(1-e_4))}$$

(5.5.32)

Using the continuity conditions (5.5.4)

$$p_o^* = \frac{\alpha n_o(e_5-e_4)+Kn_o((e_5^2-e_4^2)/2+\beta(e_5-e_4))}{((e_5-e_4)+T^{-1}(\exp(-Te_5)-\exp(-Te_4))} \quad (5.5.33)$$

and

$$CC = \frac{6(m_o+\bar{m}_o)}{(3\alpha n_o(e_5-e_4)^2-Kn_o(e_5^3-3\beta e_5^2-3e_4^2+6\beta e_4-3\beta e_4^2+2e_4^3) \\ -p_o^*(3(e_5-e_4)^2-6T^{-2}(\exp(-Te_5)-\exp(-Te_4))-6T^{-1}\exp(-Te_4)(e_5-e_4)))}$$

(5.5.34)

5.5.6 Special Case

Let $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$

Therefore $e_4 \leq \beta$, equation (5.5.20) reduces to

$$p_o^* = \frac{3\alpha(1-e_4)^2+K(1-\beta)^3}{(3(1-e_4)^2-6T^{-2}(\exp(-T)-\exp(-Te_4))-6T^{-1}\exp(-Te_4)(1-e_4))}$$

(5.5.35)

when $e_5 \leq \beta$, equations (5.5.21) and (5.5.22) reduce to

$$p_o^* = \frac{\alpha(e_5-e_4)}{(e_5-e_4) + T^{-1}(\exp(-Te_5) - \exp(-Te_4))} \qquad (5.5.36)$$

and

$$CC = \frac{4}{(\alpha(e_5-e_4)^2-p_o^*((e_5-e_4)^2 -2T^{-1}\exp(-Te_4)(e_5-e_4)-2T^{-2} -2T^{-2}(\exp(-Te_5)- \exp(-Te_4))))} \qquad (5.5.37)$$

when $\beta \leq e_5$, equations (5.5.23) and (5.5.24) will be

$$p_o^* = \frac{2\alpha(e_5-e_4) + K (e_5-\beta)^2}{2(e_5-e_4) + 2T^{-1} (\exp (-Te_5)- \exp(-Te_4))} \qquad (5.5.38)$$

and

$$CC = \frac{12}{(3\alpha(e_5-e_4)^2+K(e_5-\beta)^3-p_o^*(3(e_5-e_4)^2-6T^{-2}(\exp(-Te_5)-\exp(Te_4)) -6T^{-1}\exp(-Te_4) (e_5-e_4)))} \qquad (5.5.39)$$

when $\beta \leq e_4$, equations (5.5.32), (5.5.33) and (5.5.34) will be

$$p_o^* = \frac{3\alpha(1-e_4)^2+K(1-3\beta+6\beta e_4-3e_4^2 + 2e_4^3 - 3\beta e_4^2)}{3(1-e_4)^2-6T^{-2}(\exp(-T)-\exp(-Te_4)-6T^{-1}(\exp(-Te_4))(1-e_4)} \qquad (5.5.40)$$

$$p_p^* = \frac{2\alpha(e_5-e_4) + K (e_5^2 - e_4^2)-2K\beta (e_5-e_4)}{(e_5-e_4) + T^{-1}(\exp(-Te_5) -\exp (-Te_4))} \qquad (5.5.41)$$

and

$$CC = \frac{.2}{(3\alpha(e_5-e_4)^2+K(e_5^3-3\beta e_5^2-3e_4^2+6\beta e_4-3\beta e_4^2+2e_4^3) -p_o^*(3(e_5-e_4)-6T^{-2}(\exp(-Te_5)-\exp(-Te_4)) -6T^{-1}\exp(-Te_4)(e_5-e_4)))} \quad (5.5.42)$$

when $\alpha = \beta = 1$

$$p_o^* = \frac{(1-e_4)^2}{(1-e_4)^2-2T^{-2}(\exp(-T)-\exp(-Te_4)-2T^{-1}\exp(-Te_4)(1-e_4)} \quad (5.5.43)$$

$$p_o^* = \frac{(e_5-e_4)}{(e_5-e_4)+T^{-1}(\exp(-Te_5)-\exp(-Te_4))} \quad (5.5.49)$$

$$CC = \frac{4}{((e_5-e_4)^2-p_o^*((e_5-e_4)-2T^{-2}(\exp(-Te_5)-\exp(-Te_4)) -2T^{-1}\exp(-Te_4)(e_5-e_5)))} \quad (5.5.45)$$

5.6 CONCLUSION

Three graphs have been drawn between CC and p_o^* for different values of T, α and β. Fig. 5.6.1 is drawn for $\alpha = \beta = 1$ and for different T. It is clear from this graph that as the value of T increases, the range of first mode increases. When the critical points of all the T are joined for $\alpha = \beta = 1$, one gets a curve which seperate the mode I from mode II and the equation of this curve is

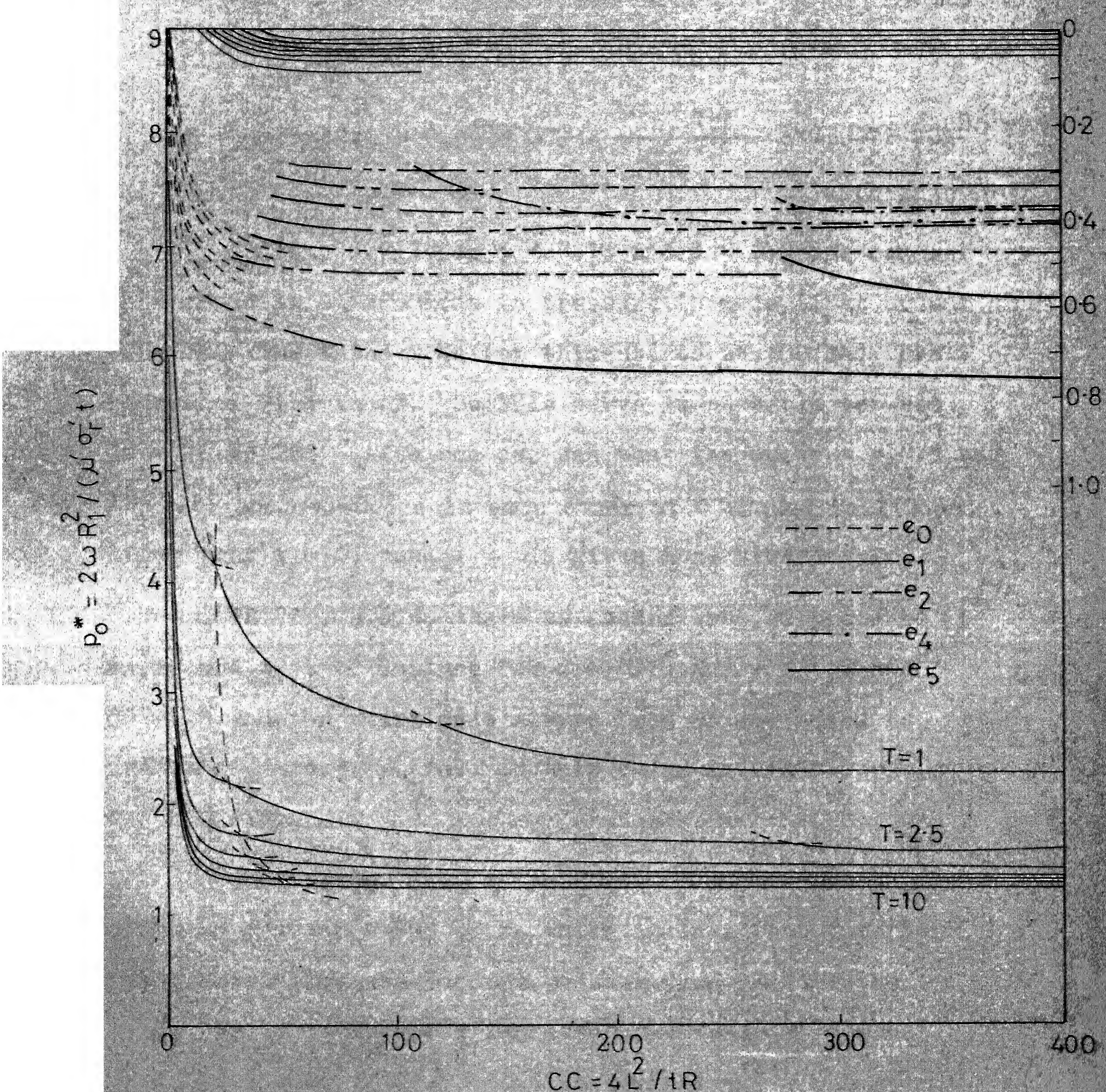


Fig. 5·6·1 p_o^* Vs CC for different T at $\alpha = \beta = 1$ for silos for Janssen's pressure distribution theory

$$p_o^* = -35.3041 + 0.8929\ CCR + \frac{500.6416}{CCR} - 0.0072\ CCR^2 \qquad (5.6.1)$$

In Fig. 5.6.2 for $\alpha = 0.75$ and $\beta = 0.5$ one can see that there is sudden drop in pressure in mode II at around CC = 76. The third mode for this starts at 288 and there is sudden rise in p_o^*. So this curve is unstable between CC = 76 to 288. Also one can see that the curve $\alpha = 0.5$ and $\beta = 0.25$ and $\alpha=\beta=0.5$ cuts each other at CC equal to 310 and after that $\alpha = 0.5$ and $\beta = 0.5$ gives more pressure.

In Fig. 5.6.3, there is sudden drop of pressure in second mode of failure for $\alpha = 0.75$ and $\beta = 0.5$ at CC = 48 and the third mode starts from 93 and there is sudden rise in pressure. So this curve is unstable in this range.

This sudden drop in pressure is due to the reason that the value of e_2 which is less than β before the drop in pressure becomes greater than β after the drop. As the value of T increases, the range of second mode for $\alpha = 0.75$ and $\beta = 0.5$ is shrinking.

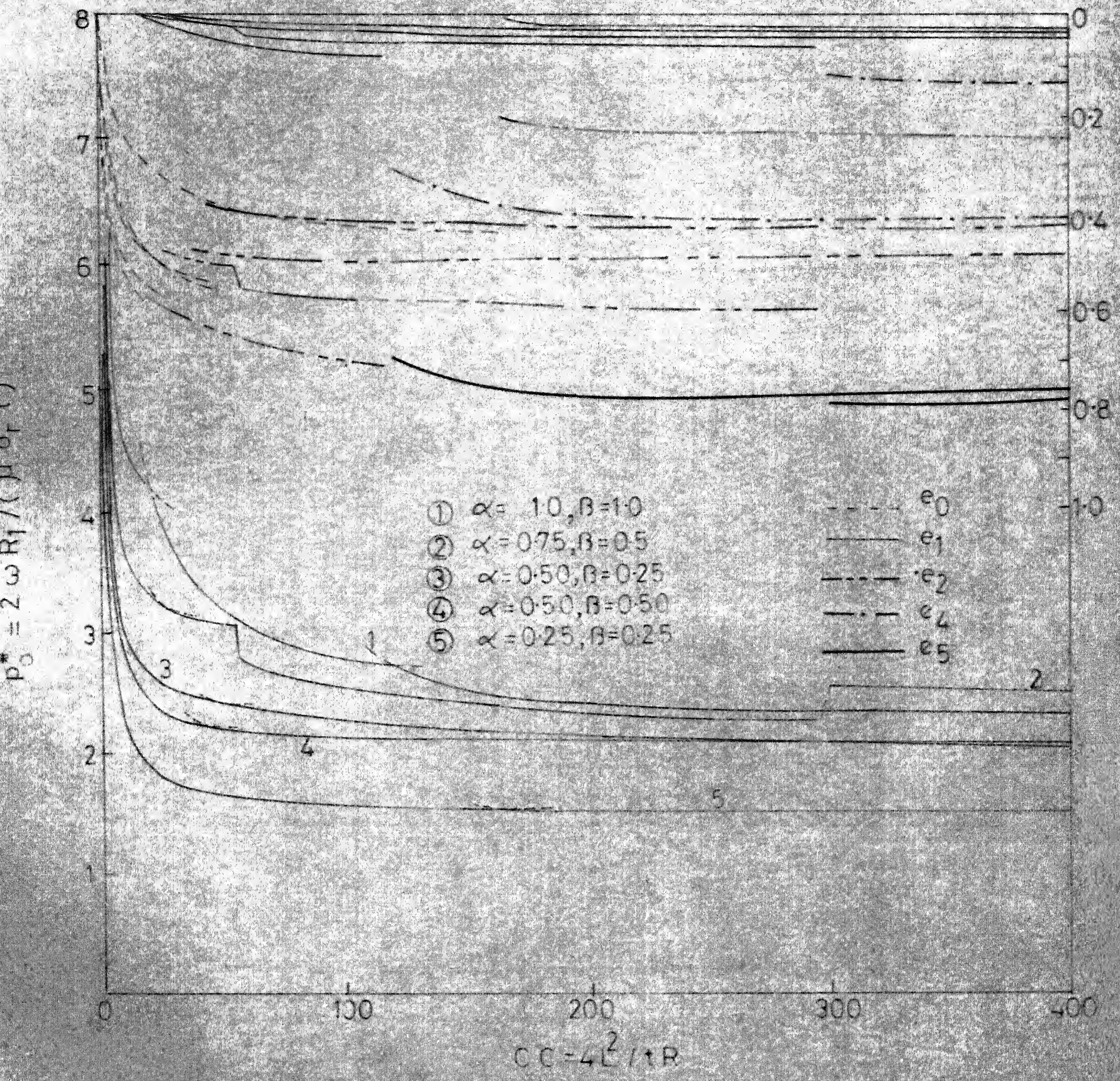


Fig. 5·6·2 p_o^* Vs CC for variable α and β at T=1 for silos for Janssen's pressure distribution theory

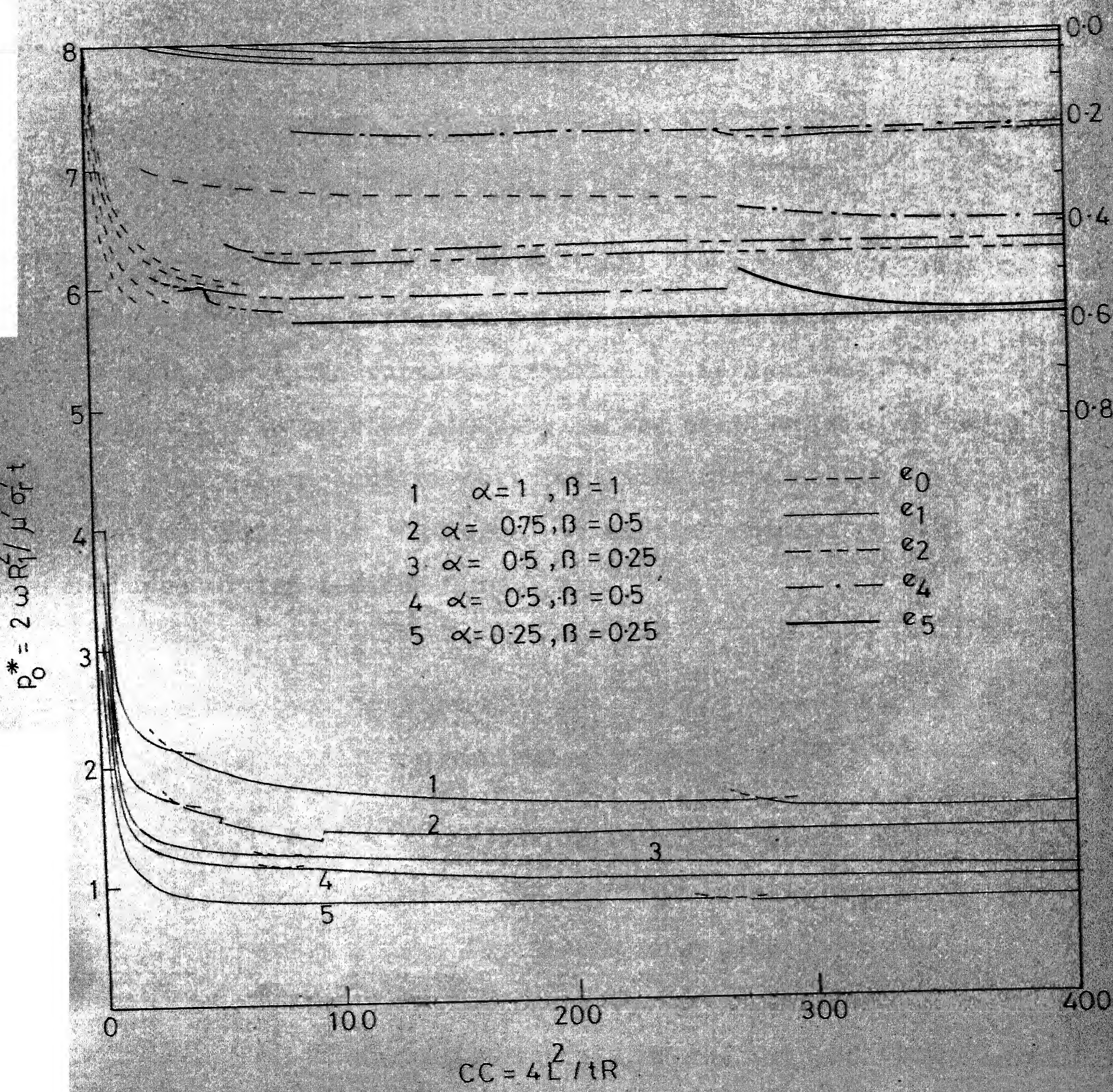


Fig. 5·6·3 p_o^* Vs CC for variable α and β at T=2·5 for silos for Janssen's pressure distribution theory

6. LIMIT DESIGN OF SILO

6.1 GENERAL

A structure is to be designed for a satisfactory performance under different forces acting on it. It should not only serve the purpose for which it is designed but it should also provide adequate safety against failure under most severe load conditions. The limit state design considers different 'failures' at service conditions and collapse states besides durability[14].

In this thesis only limit state design for strength has been considered. The working stress design is available readily in various standard books[3,13].

The pressure distribution in silos depends upon the type of material to be filled, type of silo walls, emptying condition etc. The pressure in silo during emptying is larger than the pressure during filling[6]. This increase in pressure depends upon the type of flow during emptying. There are generally two types of flow, non-dynamic and dynamic, the second flow gives larger pressure increase than the first flow. These flows depend on whether the material stored is compacted or looseprior to emptying. If the material before emptying is compacted,

it will give raise to nondynamic flow and if it is loose, then the flow will be dynamic. There is no method to prevent dynamic flow.

Here the Janssen's theory for pressure distribution in silos has been considered for design, which gives pressure at rest or for nondynamic type of flow[4].

Here the results from chapter five have been simplified for limit-state design. Only the value of M_p and N_p will change and so the value of CC.

$$M_{rb} = K\, d^2\, f_{ck} \qquad (6.1.1)$$

$$N_o = 0.87\, A_{st} f_y = 0.87\, \mu_{\theta t}\, t\, f_y \text{ , for tension} \qquad (6.1.2)$$

$$N_o = 0.4\, f_{ck}\, t(1-\mu_{\theta c}) + 0.67\, \mu_{\theta c}\, t\, f_y \text{ ,for compression} \qquad (6.1.3)$$

where

f_{ck} = characteristic compressive strength of concrete,

f_y = characteristic yield strength of steel,

d = effective thickness of shell wall,

t = total thickness of shell wall,

$\mu_{\theta t}$ = ratio of steel to concrete for circumferential tension,

$\mu_{\theta c}$ = ratio of steel to concrete for circumferential compression,

K = depends upon the type of steel and concrete.

It has been assumed in analysis that $\mu_{\theta t} = \mu_{\theta c} = \mu_\theta$, to simplify the equation and also because of the reason that the compression zone is very small and confined to 0.1 L of the upper portion.

Now considering the equilibrium equation

$$\frac{d^2M_x}{dx^2} + \frac{N_\theta}{R} = 1.5\ p \qquad (6.1.4)$$

where 1.5 is partial safety factor for static condition.

Now

$$m_x = \frac{M_x}{M_{rb}}$$

$$n_\theta = \frac{N_\theta}{N_o}$$

$$p^* = \frac{pR}{N_o} \qquad (6.1.5)$$

Substituting these values in equation (6.1.4)

$$\frac{Kd^2\ f_{ck}\ R}{L^2} \ \frac{d^2m_x}{de^2} + n_\theta(0.87\mu_\theta t\ f_y) = 1.5p^*(0.87\mu_\theta t\ f_y)$$

$$\frac{Kd^2f_{ck}\ R}{L^2(0.87\ \mu_\theta t\ f_y)} \ \frac{d^2m_x}{de^2} + n_\theta = 1.5P^*$$

$$\frac{d^2 m_x}{de^2} + CC\ n_\theta = 1.5\ CC\ p^* \qquad (6.1.6)$$

where

$$CC = \frac{(0.87\ \mu_\theta\ t\ f_y)\ L^2}{Kd^2\ f_{ck}\ R}$$

and

$$e = \frac{X}{L} \qquad (6.1.7)$$

6.2 NORMALISATION OF JANSSEN'S EQUATIONS AND YIELD CONDITION FOR SILO

6.2.1 Equilibrium Equation

Consider the equilibrium equation (6.1.6)

$$\frac{d^2 m_x}{de^2} + CC\ n_\theta = 1.5\ CC\ p^* \qquad (6.1.6)$$

where

$$p^* = \frac{2\gamma R_1^2}{\mu'\ (0.87\,\mu_\theta t\ f_y\ )}\ (1-\exp(-Te)),$$

$$T = \mu'\ k\ L\ /R_1$$

and

μ' = coefficient of friction between stored material and silo wall,

k = ratio of horizontal static pressure to vertical static pressure due to stored material,

R_1 = hydraulic radius of the silo

γ = unit weight of stored material

Now

$$p^* = 1.5\, p_o^* \;(1 - \exp(-Te)) \qquad (6.2.1)$$

where

$$p_u^* = 1.5\, p_o^* = 3\gamma R_1^2/\;(0.87\mu_\theta t\, f_y)$$

So equation (6.1.6) will be

$$\frac{d^2_{mx}}{de^2} + CC\, n_\theta = CC\, p_u^*\,(1-\exp(-Te)) \qquad (6.2.2)$$

All the other equations and results of chapter five will remain the same for all modes because the initial equation is same. Here only the results for the case of $\alpha = \beta = 1$ have been listed for the design purpose.

6.2.2 Results for Mode I

When $m_o = \bar{m}_o = \bar{n}_o = 1$ and $n_o = \beta\mu_x$

$$m_{x1} = CC(p_u^*(e^2/2-T^{-2}\exp(-Te)-T^{-1}e+T^{-2})-\alpha n_o e^2/2) \qquad (6.2.3)$$

$$p_u^* = (6/CC + 3n_o)/(3(1+2T^{-2}(1-\exp(-T))-2T^{-1}) \qquad (6.2.4)$$

$$p_u^* = n_o e_o\;/(e_o+T^{-1}\exp(-Te_o)-T^{-1}) \qquad (6.2.5)$$

and

$$CCR = 2/(n_o e_o^2-p_u^*(e_o+2T^{-2}(1-\exp(-Te_o))-2T^{-1}e_o) \qquad (6.2.6)$$

6.2.3 Results for Mode II

When $m_o = \bar{m}_o = \bar{n}_o = 1$ and $n_o = \beta\ \mu_x$

For $e \leq e_1$

$$m_{x1} = CC(p_{\ddot{u}}^{*}(e^2/2+T^{-2}(1-\exp(-Te))-T^{-1}e)+n_o e^2/2) \quad (6.2.7)$$

For $e \geq e_1$

$$m_{x2} = CC\ (p_{\ddot{u}}^{*}(e^2/2+T^{-2}(1-\exp(-Te))-T^{-1}e)-n_o e^2/2 + (ee_1-e_1^2/2)\ (1+n_o)) \quad (6.2.8)$$

$$p_{\ddot{u}}^{*} = \frac{2/CC + n_o-(1+n_o)(2e_1-e_1^2)}{(1+2T^{-2}(1-\exp(-T))-2T^{-1}\ )} \quad (6.2.9)$$

$$p_{\ddot{u}}^{*} = \frac{n_o e_2 - e_1\ (1+n_o)}{e_2+T^{-1}(\exp(-Te)-1)} \quad (6.2.10)$$

$$CC = \frac{2}{(n_o e_2^2-(1+n_o)(2e_1e_2-e_1^2)-p_{\ddot{u}}^{*}(e_2^2-2T^{-2}\exp(-Te_2) -2T^{-1}e_2+2T^{-2}))} \quad (6.2.11)$$

$$p_{\ddot{u}}^{*} = \frac{n_o(e_2-e_3)}{((e_2-e_3) + T^{-1}(\exp(-Te_2)-\exp(-Te_3))} \quad (6.2.12)$$

$$CCR = \frac{2}{(p_{\ddot{u}}^{*}(e_3^2+2T^{-2}(1-\exp(-Te_3))-2T^{-1}e_3)-n_o e_3^2 +(1+n_o)(2e_1e_3-e_1^2))} \quad (6.2.13)$$

6.2.4 Results for Mode III

When $m_o = \bar{m}_o = \bar{n}_o = 1$ and $n_o = \beta\mu_x$

$$m_x = CC(p_u^*(\frac{(e-e_4)^2}{2} - T^{-1}\exp(-Te_4)(e-e_4) - T^{-2}(\exp(-Te)-\exp(-Te_4))$$

$$-n_o/2\ (e-e_4)^2) + 1 \qquad (6.2.14)$$

$$p_u^* = \frac{n_o(1-e_4)^2}{((1-e_4)^2 - 2T^{-2}(\exp(-T)-\exp(-Te_4)) - 2T^{-1}\exp(-Te_4)(1-e_4))} \qquad (6.2.15)$$

$$p_u^* = \frac{n_o(e_5 - e_4)}{(e_5-e_4)+T^{-1}(\exp(-Te_5)-\exp(-Te_4))} \qquad (6.2.16)$$

$$CC = \frac{4}{(n_o(e_5-e_4)^2 - p_u^*((e_5-e_4)^2 - 2T^{-1}\exp(-Te_4)(e_5-e_4) - 2T^{-2}(\exp(-Te_5)-\exp(-Te_4)))} \qquad (6.2.17)$$

6.2.5 Yield Condition

The yield condition for the case of silos is

$$m_x(1+\alpha)+2n_x^2+2n_x(2\beta\mu_x+\alpha-1)+2\beta^2\mu_x^2-4\beta\mu_x-2\alpha = 0, \text{ for } m_x > 0$$

and

$$-m_x(1+\alpha)+2n_x^2+2n_x(\alpha-1)-2\alpha=0 \quad \text{for} \quad m_x < 0.$$

This cylinder is bounded by the planes with equations.

$n_\theta = 1$ for compression

$n_\theta = -\alpha - \beta\mu_\theta$ for tension ,

where

$$n_x = \frac{N_x}{0.4f_{ck}t(1-\mu_{xc}) + 0.67\mu_{xc}\ t\ f_y} \quad \text{for compression}$$

$$n_x = \frac{N_x}{0.87\mu_{xt}\ f_y} \quad , \quad \text{for tension}$$

$$m_x = \frac{M_x}{Kd^2\ f_{ck}}$$

$$n_\theta = \frac{N_\theta}{0.87\mu_{\theta t}\ f_y} \quad \text{for tension,}$$

$$n_\theta = \frac{N_\theta}{0.4f_{ck}t(1-\mu_{\theta c})+0.67\mu_{\theta c}t\ f_y} \quad \text{for compression .}$$

Assume that

$\mu_x = A_x/t$, reinforcement ratio in axial direction,

$\mu_{\theta c} = \mu_{\theta t} = \mu_\theta = A_\theta/t$, reinforcement ratio in circumferential direction,

$$\alpha = \frac{\text{tensile yield strength of concrete}}{\text{compressive yield strength of concrete}} ,$$

$$\beta = \frac{\text{yield strength of reinforcement}}{\text{compressive yield strength of concrete}} .$$

6.2.6 Interaction Equations

The interaction equations for combined bending and axial force for balance force pair, i.e. the section fails simultaneously in compression and in tension, are

$$P_{ab} = 0.36\, k_u\, b\, d\, f_{ck} + 0.87\, (A_{sc} - A_{st}) \qquad (6.2.18)$$

$$P_{ab} e_s = K b\, d^2\, f_{ck} + 0.87\, A_{sc}\, D_s\, f_y \qquad (6.2.19)$$

$$P_{ao} = (1-\mu_x)\,(0.45\, b\, t\, f_{ck}) + 0.75\, f_y\, b\, t\, \mu_x \qquad (6.2.20)$$

$$e_x = M_x / N_x$$

$$P_a = \frac{P_{ao}}{1 + (P_{ao}/P_{ab} - 1)\, e_x/e_b} \qquad (6.2.21)$$

If $N_x < P_a$ then the design is adequate[14].

where

e_s = distance of P_{ab} from tension steel ,

= $0.5\, D_s + e_b$,

D_s = distance between compression and tension steel,

e_b = distance of P_{ab} from centre of the shell wall,

A_{sc} = area of steel in compression,

A_{st} = area of steel in tension,

k_u = depends upon the type of steel,

e_x = actual eccentricity,

μ_x = ratio of area of axial reinforcement to concrete,

P_a = actual axial capacity of the silo wall along with moment,

P_{ao} = axial load capacity with zero eccentricity,

P_{ab} = axial load capacity with eccentricity e_a.

6.3 DESIGN OF SILO

6.3.1 General

Here, design is based on static pressure distribution only. No correction factor has been considered for wind force, earthquake force or emptying condition.

DATA:

Material stored	:	Cement
Concrete mix	:	M20
Steel	:	Grade Fe 415
Unit weight of cement	:	14.1 kN/m^3
Ratio of horizontal static pressure to vertical static Pressure due to stored cement	:	0.27
Coefficient of friction between cement and silo wall	:	0.70
Minimum reinforcement	:	0.25 percent

Short silo	Diameter :	1.5m	Height	: 5 m
Medium silo	Diameter :	3.0m	Height	:10 m
Deep silo	Diameter :	6.0m	Height	:20 m

Axial pressure due to friction is

$$N_x = L \int_0^e \gamma R_1 (1-\exp(-Te))\, de = \gamma R_1 L (e + T^{-1}(\exp(-Te)-1)) \quad (6.3.1)$$

where L = total depth of silo

So

$$T = 2.52$$

$$CC = 130.815 \left(\frac{\mu_\theta t\, L^2}{d^2 R} \right)$$

$$\alpha = 0.0$$

$$\beta = 20.75$$

Now the yield condition will be

$$2n_x^2 + 2n_x(41.5\mu_x - 1) + 861.125\mu_x^2 - 83\mu_x + m_x = 0$$

$$\text{for } m_x > 0 \qquad (6.3.2)$$

$$2n_x^2 - 2n_x - m_x = 0 \quad \text{for } m_x < 0 \qquad (6.3.3)$$

and $n_\theta = 1$ for compression

$n_\theta = -20.75\ \mu_\theta$ for tension

Moment distribution for first mode, second mode and third mode have been shown in Fig. 6.3.1 for T = 2.52 and $m_o = \bar{m}_o = n_o = \bar{n}_o = 1$ for CC = 20, 130 and 525 respectively.

6.3.2 Design of Short Silo

6.3.2.1 Design of wall

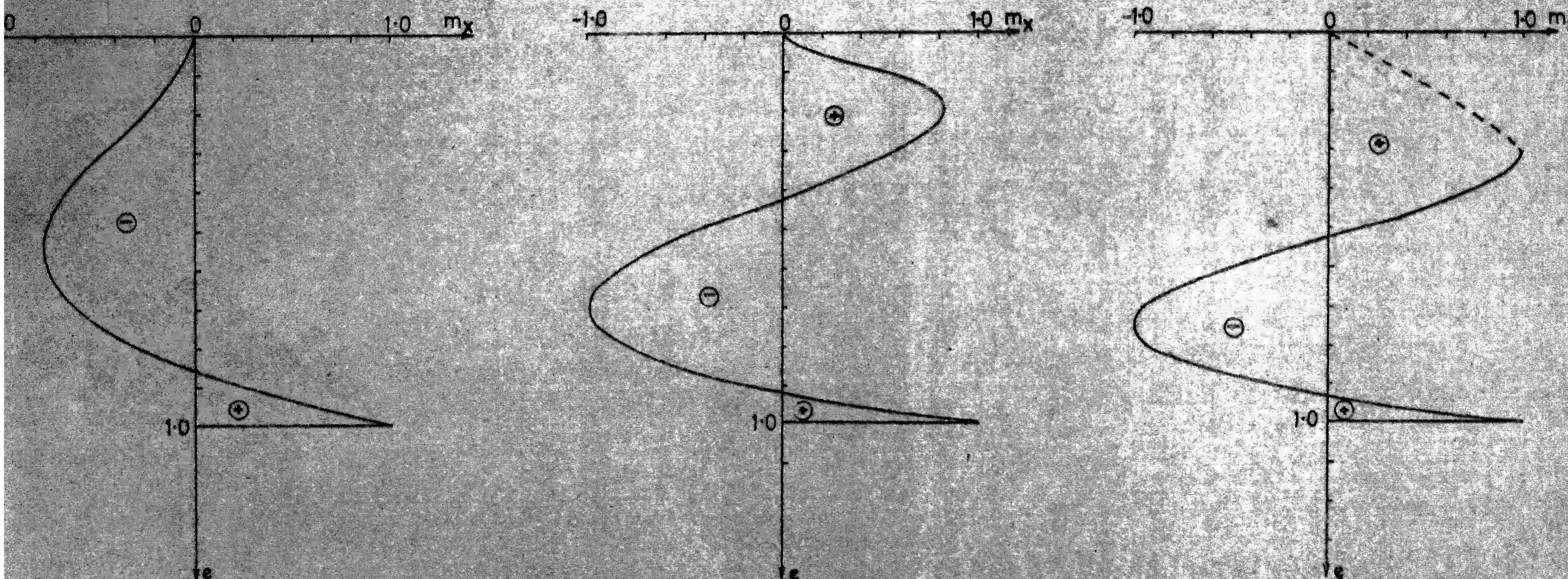
Let,

Percentage of circumferential reinforcement	:	0.25
Total thickness of silo wall	:	0.12 m
Effective thickness of silo wall	:	0.10 m

So

$$CC = 130.815$$

and $T = 2.52$



(a) First mode for CC=20 (b) Second mode for CC=130 (c) Third mode for CC=525

Fig.6·3·1 Moment distribution in silos wall for Janssen's pressure distribution theory for $n_o = \bar{n}_o = m_o = \bar{m}_o = 1$ and $T = 2{\cdot}52$

By using equations (6.2.3) through (6.2.6)

$$p_u^* = 0.1354$$

and $p_u^* = 3\gamma R_1^2 / (0.87\,\mu_\theta t\ b\ f_y\,\mu')$

From this $\mu_\theta = 0.145$ percent.

Minimum required reinforcement is 0.25 percent and provided is also the same. So circumferential reinforcement is adequate. Now for all m_x, n_x is calculated for known n_θ from equations (6.3.2) and (6.3.3). The required n_x is calculated from equations (6.1.3) and (6.3.1). All these stress resultants n_θ, m_x, n_x and also required n_x for this case have been presented in Fig. 6.3.2 for $\mu_x = 1.75$ percent and $\mu_\theta = 0.25$ percent in the complete silo.

Now, $M_{xu} = 1.5\ K\ b\ d^2\ f_{ck} = 41.4$ kNm/m

where, $K = 0.138$ for the given steel.

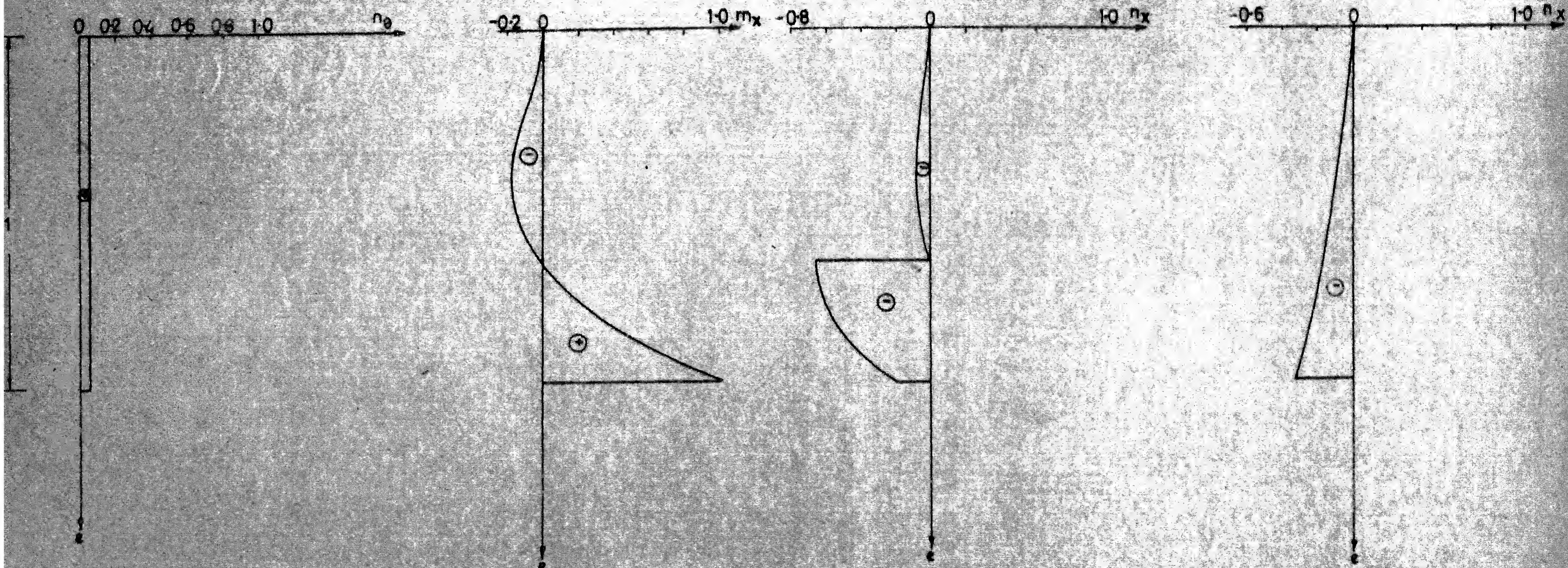
For this case, critical situation is at bottom of the silo because of the maximum positive moment and the maximum axial force.

At $e = 1$

$N_x = 47.7$ kN/m

and $M_x = M_{xu} = 41.4$ kNm/m

M_x is positive at this level so n_x calculated from yield condition as well as n_x required will depend upon μ_x.



(a) Hoop force (b) Bending moment (c) n_x from yield condition (d) n_x from equilibrium equation

Fig. 6·3·2 Stress distribution for first mode for $\mu_x = 1{\cdot}75\,\%$ and $\mu_\theta = 0{\cdot}25\,\%$

Provided μ_x = 1.75 percent at bottom, out of which μ_{xt} = 1.25 percent and μ_{xc} = 0.5 percent. So from equations (6.2.18) through (6.2.21).

P_{ab} = 88.911 kN/m

$P_{ab}\,e_s$ = 44.93 kNm/m

e_b = 0.465 m

e_x = 0.868 m

P_a = 48.88 kN/m > 47.7 kN/m so safe.

The other critical point is at e =0.66. Here the calculated n_x is less than the required n_x. The moment is changing from positive to negative. If the μ_x is changed , n_x calculated will not change except n_x required. Here,

M_x = 0.013175 M_{xu} = 0.545 kNm/m

N_x = 28.27 kN/m

Alternate bars from outer face and alternate two bars from inner face have been curtailed from e = 0.75 onward. So, μ_{xt} = 0.25 percent and μ_{xc} = 0.4166 percent.

Now from equations (6.2.18) through (6.2.21)

P_{ab} = 486.037 kN/m

$P_{ab}e_s$= 35.139 kNm/m

e_b = 0.0323 m

P_{ao} = 1356.42 kN/m

e_x = 0.0193 m

P_a = 655.44kN/m > 28.27 kN/m so safe.

Now from e = 0.5 onward alternate bars from inner face have been curtailed, so the reinforcement at e = 0.43 where the maximum negative moment occurs is

μ_{xt} = 0.25 percent and μ_{xc} = 0.2083 percent,

M_x = 0.167 M_{xu} = 6.92 kNm/m and N_x = 16.315 kN/m.

From equations (6.2.18) through (6.2.21).

P_a = 26.7 kN/m > 16.315 kN/m. So safe

Reinforcement details:

Provide 8 Ø bars at 300 C/C as circumferential reinforcement on both faces. Provide 10 Ø bars at 50 C/C as axial reinforcement on inner face and at 130 C/C outer face from e = 1.0 to 0.75. Provide 10 Ø bars at 150 C/C on inner face and 260 C/C on outer face from e = 0.75 to 0.5. From e = 0.5 onward provide 10 Ø bars at 300 C/C on inner face and 260 C/C on outer face.

6.3.2.2 Design of conical dome and ring beam

Data:

Diameter of opening provided in conical dome	:	0.25 m
Angle of inclination of conical dome wall from vertical axis	:	45^o
Thickness of the wall	:	0.12 m

P_u = 47.7 kN/m

H_1 = $P_u \tan\theta$ = 47.7 kN/m

H_2 = 1.5p = 10.425 kN/m^2

where

P_u = vertical force due to wall and stored cement,

H_1 = radial force due to the action of vertical force,

H_2 = radial cement pressure,

θ = angle of inclination of cone from vertical axis, and

b = width of ring beam.

Let b = 0.2 m

Total hoop tension = D/2 (H_1 + H_2 b) = 37 kN

So steel required in ring beam = 102.5 mm^2

Provide two 10ϕ bars in ring beam excluding circumferential reinforcement.

Slanted length of the conical dome	=	0.884 m
Diameter at mid height	=	0.875 m
W_g(weight of conical hopper)	=	7.3 kN

W_m(weight of material stored in conical hopper) = 6.2 kN

$$q_{des} = \frac{\gamma R_1}{k} \left(1 - \exp(-T)\right)$$

where q_{des} = vertical static pressure at the top of the hopper. Substituting the values, one gets

$$q_{des} = 25.75 \text{ kN/m}^2$$

$$F_{mu} = 1.5 \left(\frac{q_{des} D}{4 \sin \theta} + \frac{W_m}{\pi D \sin \theta} + \frac{W_g}{\pi D \sin \theta} \right)$$

where, F_{mu} = diagonal force per unit length, and D = diameter of the hopper at the level being considered.

F_{mu} (at top of the hopper) = 26.56 kN

F_{mu} (at middle of the hopper) = 22.32 kN

F_{mu} (at bottom of the hopper) = 39.88 kN.

So the required meridional reinforcement at bottom,

A_s = 110.5 mm^2/m.

This is less than the minimum reinforcement, so provide minimum reinforcement i.e. 0.25 percent. So provide 8 Ø bars at 165 C/C at outer face only. Curtail alternate bars from the point where the diameter reduces to 0.75 m.

$$F_{tu} = \frac{1.5\, q_{des} D}{2 \sin \theta}$$

where F_{tu} = hoop force per unit width.

Substituting the values, one gets

F_{tu} = 41 kN/m

So the required hoop reinforcement, A_s = 113.47 mm^2/m.

This is less than the required minimum, so provide minimum required reinforcement i.e. 0.25 percent. Provide 8 Ø bars at 165 C/C at outer face only.

The reinforcement details are shown in Fig. 6.3.5.

6.3.2.3 Cost computation

Unit costs of the materials including fixing, or placing or finishing etc. are [14].

One m^3 of M20 concrete	= Rs. 450.00
One m^3 of steel	= Rs. 3354.0
Volume of concrete in wall and ring beam	= 2.928 m^3
Volume of concrete in hopper	= 0.292 m^3
Total volume of concrete	= 3.22 m^3
Cost of concrete	= Rs. 1450.00
Volume of steel on wall and ring beam	= 0.0321 m^3
Volume of steel in hopper	= 0.00161 m^3
Total volume of steel	= 0.03371 m^3
Cost of steel	= Rs. 1131.00
Total cost	= Rs. 2581.00
Total stored cement	= 9.275 m^3

Cost of storing for one cubic meter of cement = Rs. 278.28.

6.3.3 Design of Medium Silo

6.3.3.1 Design of wall

Let ,

Percentage of circumferential reinforcement	: 0.5
Total thickness of silo wall	: 0.12 m
Effective thickness of silo wall	: 0.10 m

So, CC = 523.26 and T = 2.52.

By using equations (6.2.7) through (6.2.13)

$p_u^* = 0.18453$, so $\mu_\theta = 0.43$ percent. Provided is 0.5 percent, so the circumferential reinforcement is adequate. Now for all m_x, n_x is calculated for known n_θ from equations (6.3.2) and (6.3.3). Also the required n_x is calculated from equations (6.1.3) and (6.3.1). All these stress resultants n_θ, m_x, n_x and also required n_x for this case have been shown in Fig. 6.3.3 for μ_θ=0.5 percent and $\mu_x = 1.75$ percent throughout.

$M_{xu} = 1.5\ K\ b\ d^2\ f_{ck} = 41.4$ kNm/m.

For this case, critical situation is at bottom of the silo because of the maximum positive moment and the maximum axial force. Now at $e = 1$, $N_x = 145.7$ kN/m, $_{xu} = 41.4$ kNm/m.

The reinforcement provided on inner face is 1.25 percent and on outer face is 0.5 percent. So $\mu_{xt} = 1.25$ percent and $\mu_{xc} = 0.5$ percent . So from equations (6.2.18) through (6.2.21).

$P_a = 146$ kN/m > 145.7 kN/m. So safe.

At $e = 0.89$, n_x calculated from yield condition is smaller than the required. This is because moment is changing from positive to negative. Here the tension face is becoming compression face and compression face is becoming tension face so , $\mu_{xt} = 0.5$ percent, $\mu_{xc} = 1.25$ percent.

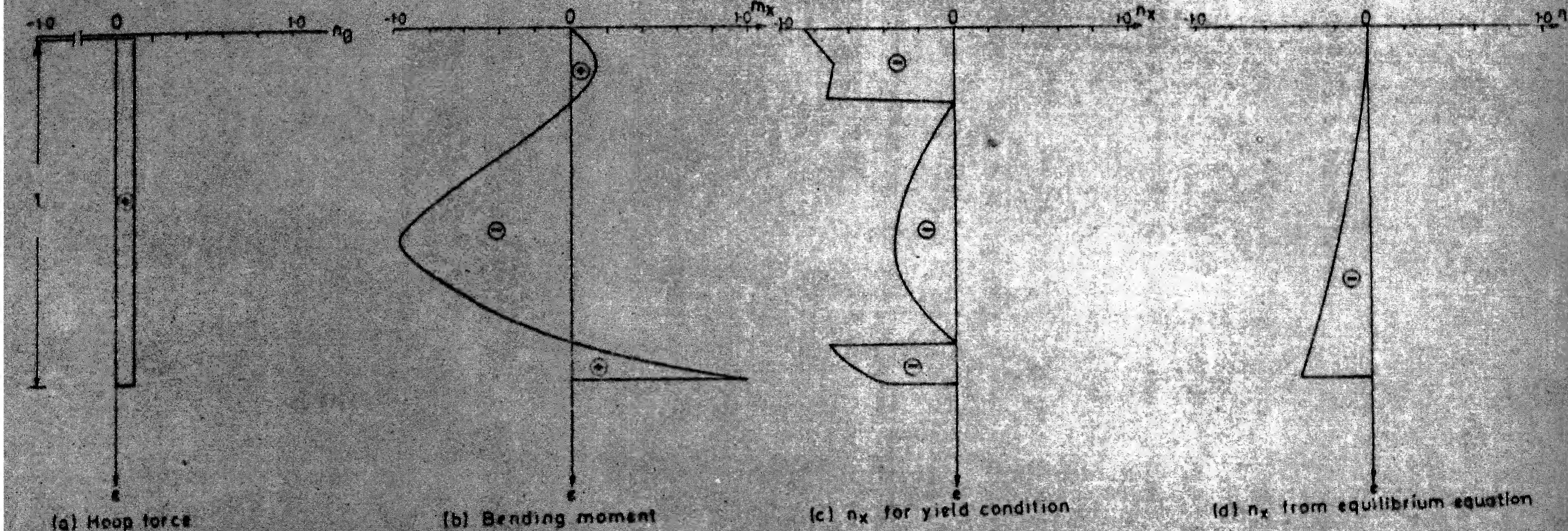


Fig. 6·33 Stress distribution for second mode for $\mu_x = 1·75\%$ and $\mu_\theta = 0·5\%$

Now from equations (6.2.18) through (6.2.21)

P_a = 1628 kN/m > 126.8 kN/m , so safe.

At e = 0.61 the negative moment is maximum i.e. M_{xu} and N_x is equal to 74.8 kN/m. From equations (6.2.18) through (6.2.21)

P_a = 80.6 kN/m > 74.8 kN/m.

Here μ_{xt} = 1 percent and μ_{xc} = 0.625 percent. This is because from e = 0.8 alternate bars from inner face have been curtailed and alternate bars have been provided on outer face.

At e = 0.21 the n_x calculated is less than the n_x required. Here μ_{xc} = 0.3125 percent and μ_{xt} = 0.5 percent because alternate bars from both face have been curtailed from e = 0.3 onward. From equations (6.2.18) through (6.2.21)

P_a = 144.3 kN/m > 17 kN/m. So safe.

Now curtail alternate bars from outer face from e = 0.15 onward. At e = 0.1 there is relative maximum moment. From equations (6.2.18) through (6.2.21), one gets

P_a = 32.9 kN/m > 6.4 kN/m. So safe

Reinforcement details:

Provide 8 Ø bars 165 C/C as circumferential reinforcement on both faces. Provide 10 Ø bars at 50 C/C as axial

reinforcement on inner face and at 130 C/C on outer face from e = 1.0 to 0.8. Provide 10 ∅ bars at 100 C/C on inner face and 65 C/C on outer face from e = 0.8 to 0.3. Provide 10 ∅ bars at 200 C/C on inner face and 130 C/C on outer face from e = 0.3 to 0.15. From e = 0.15 to top provide 10 ∅ bars at 200 C/C on inner face and 260 C/C on outer face.

6.3.3.2 Design of conical dome and ring beam

Data:

Diameter of opening provided in conical dome : 0.35 m

Angle of inclination of conical dome wall from vertical axis : 45°

Thickness of the wall : 0.12 m

$P_u = 145.75$ kN/m

$H_1 = P_u \tan\theta = 145.75$ kN/m

$H_2 = 1.5\ p = 20.8$ kN/m^2

Let b = 0.2 m

Total hoop tension = 225 kN

So steel required in ring beam = 623.2 mm^2

Provide six 12 ∅ bars in ring beam excluding circumferential reinforcement.

Slanted length of the conical dome = 1.88 m

Diameter at mid height = 1.675 m

W_g (weight of conical hopper) =29.7 kN

W_m (weight of material stored in conical hopper) = 50 kN.

$q_{des} = 51.5 \text{ kN/m}^2$

F_{mu} (at top of the hopper) $= 100$ kN/m

F_{mu} (at middle of the hopper)$=$ 79.9 kN/m

F_{mu} (at bottom of the hopper)$=$ 163.26 kN/m

Reinforcement required at top of the hopper = 276.97 mm^2/m.

This is less than the minimum requirement, so provide minimum requirement of steel i.e. 10 Ø bars at 260 C/C on outer face only. Reinforcement required at bottom of the hopper is 452.2 mm^2/m. So needed reinforcement is 10 Ø bars at 170 C/C on outer face only. Now provide 10 Ø bars at 260 C/C at top of the hopper. Curtail alternate bars from the place where diameter of hopper reduces to 2 m and again alternate bars from the place where diameter becomes 1 m. So spacing of 10 Ø bars at bottom is 120 C/ C.

Now F_{tu} = 164 kN/m , so the required hoop reinforcement is 453.95 mm^2/m. Provide 10 Ø bars at 170 C/C at outer face as hoop reinforcement. For reinforcement details see Fig. 6.3.6.

6.3.3.3 Cost computation

Volume of concrete in wall and ring beam	= 11.602 m^3
Volume of concrete in hopper	= 1.188 m^3
Total volume of concrete	= 12.79 m^3
Cost of concrete	= Rs.5756

Volume of steel in wall and ring beam	=	0.2232 m^3
Volume of steel in hopper	=	0.009504 m^3
Total volume of steel	=	0.2327 m^3
Cost of steel	=	Rs. 7805
Total cost	=	Rs. 10561
Total stored cement	=	74.232 m^3
Cost of storing for one cubic meter of cement	=	Rs. 142.27

6.3.4 Design of A Deep Silo

6.3.4.1 Design of wall

Let,

Percentage of circumferential reinforcement	:	1.0
Total thickness of silo wall	:	0.12 m
Effective thickness of silo wall	:	0.10 m

So CC = 2093.04 . By using equations (6.2.14) through (6.2.17), one gets

$p_u^* = 0.3205$, so $\mu_\theta = 0.979$ percent. Provided is 1 percent, so the circumferential reinforcement provided is adequate. Now for all m_x n_x is calculated for known n_θ from equations (6.3.2) and (6.3.3). The required n_x is calculated from equations (6.1.3) and (6.3.1). All these stress resultants n_θ, m_x, n_x and also required n_x for this case have been shown in Fig. 6.3.4 for $\mu_x = 2$ percent and $\mu_\theta = 1$ percent in the complete silo.

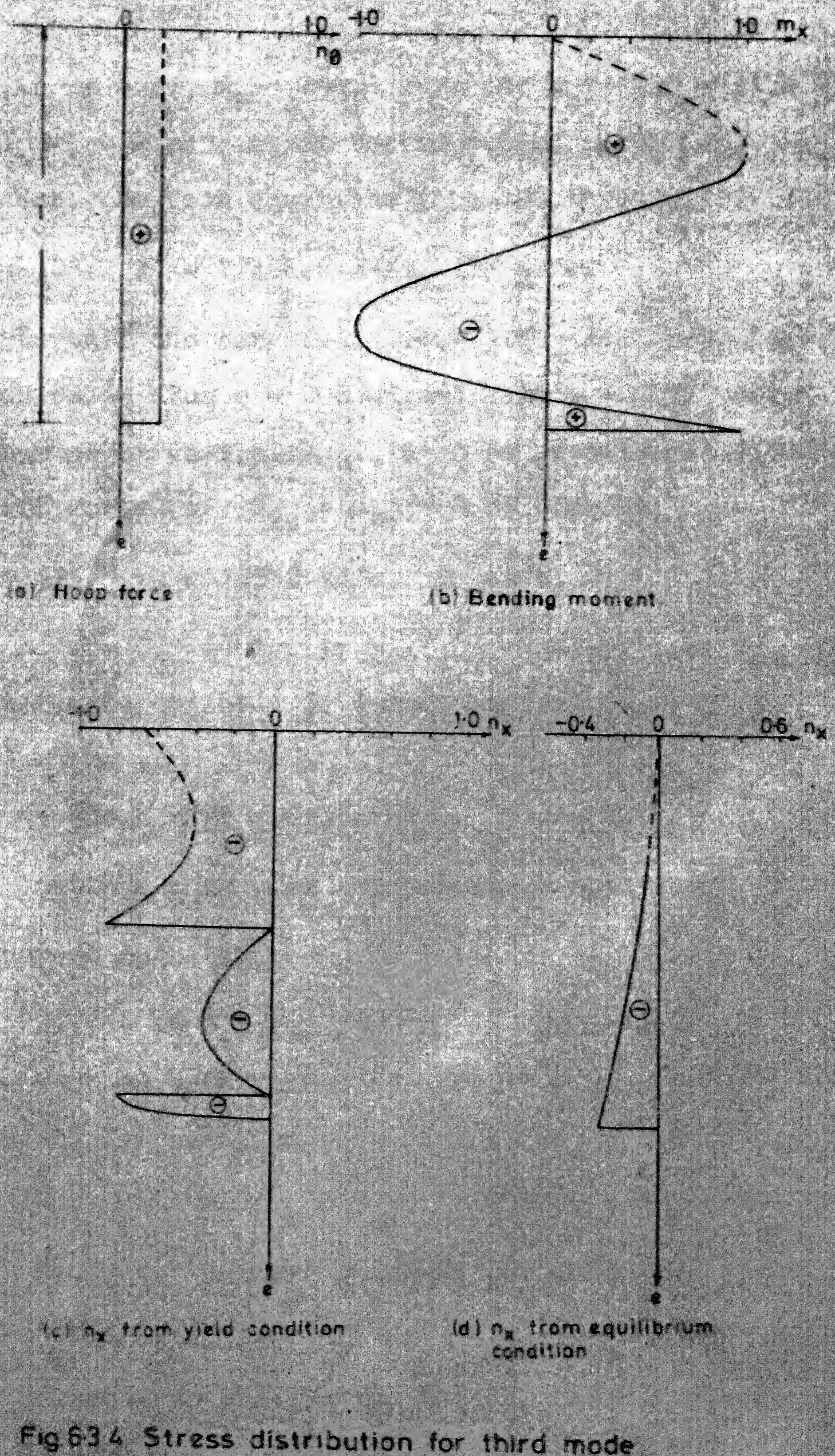


Fig.6.3.4 Stress distribution for third mode for $\mu_x = 2\%$ and $\mu_\theta = 1\%$

At $e = 1$, $N_x = 496.6$ kN/m and $M_x = M_{xu} = 41.4$ kNm/m. Provided $\mu_x = 2$ percent at bottom out of which $\mu_{xc} = \mu_{xt}$ = 1 percent. So from equations (6.2.18) through (6.2.21).

$P_a = 508.7$ kN/m > 496.6 kN/m. So safe.

Curtail one bar after every alternate three bars from both faces from e = 0.8 onward. At e = 0.75 moment is maximum negative i.e. M_{xu}. So from equation (6.2.18) through (6.2.21) for $\mu_{xc} = \mu_{xt} = 0.75$ percent.

$P_a = 425.5$ kN/m > 332 kN/m. So safe.

At e = 0.3 moment is maximum positive i.e. M_{xu}, continue the same reinforcement. So from equations (6.2.18) to (6.2.21).

$P_a = 134.2$ kN/M > 83.36 kN/m. So safe.

Continue this same reinforcement upto top.

Reinforcement details:

Provide 10 ∅ bars, 130 C/C as circumferential reinforcement on both faces. Provide 10 ∅ bars 65 C/C as axial reinforcement on both faces from e = 1.0 to 0.8. Provide 10 ∅ bars 85 C/C on both faces from e = 0.8 to top.

6.3.4.2 Design of conical dome and ring beam

Data:

Diameter of opening provided in conical dome : 0.50 m

Angle of inclination of conical dome wall from vertical axis : 45^{o}

Thickness of wall : 0.12 m

P_u = 496.6 kN/m

H_1 = 496.6 kN/m

H_2 = 41.7 kN/m^2

Let , b = 0.30 m.

Total hoop tension = 1516 kN

So steel required in ring beam = 4200 mm^2/m.

Provide 9 bars of 25 Ø in ring beam excluding circumferential reinforcement.

Slanted length of the conical dome = 3.89 m

Diameter at mid height = 3.25 m

W_g = 119.2 kN

W_m = 398.5 kN

q_{des} = 103 kN

F_{mu}(at top of the hopper) = 386 kN

F_{mu}(at middle of the hopper) = 285 kN

F_{mu}(at bottom of the hopper) = 726.5 kN

Reinforcement required at top is 1069.1 mm^2/m. So provide 12 Ø bars at 100 C/C. Reinforcement required at bottom is 2012.2 mm^2/m.

Provide 12 Ø bars at 100 C/C at top. Curtail alternate bars from the place where diameter becomes 4m and again

curtail alternate bars from the point where diameter reduces to 2 m. At bottom spacing is 34 C/C. So safe.

F_{tu} = 655.5 kN/m , so hoop reinforcement required is 1815.5 mm^2/m. So provide 12 ∅ bars at 120 C/C at outer face as hoop reinforcement. For reinforcement details see Fig. 6.3.7.

6.3.4.3 Cost computation

Volume of concrete in wall and ring beam	= 46.398 m^3
Volume of concrete in hopper	= 4.768 m^3
Total volume of concrete	= 51.166 m^3
Cost of concrete	= Rs. 23025
Volume of steel in wall and ring beam	= 1.28962 m^3
Volume of steel in hopper	= 0.07947 m^3
Total volume of steel	= 1.36909 m^3
Cost of steel	= Rs. 45920
Total cost	= Rs. 68945
Total stored cement	= 593.7484 m^3

Cost of storing for one cubic meter of cement=Rs. 116.12.

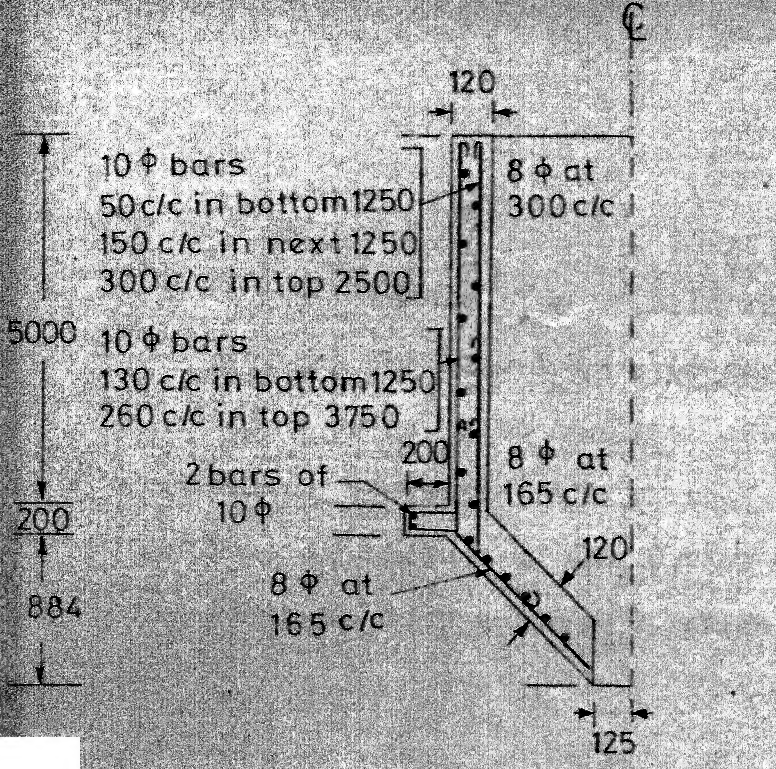


Fig.6·3·5 Reinforcement details

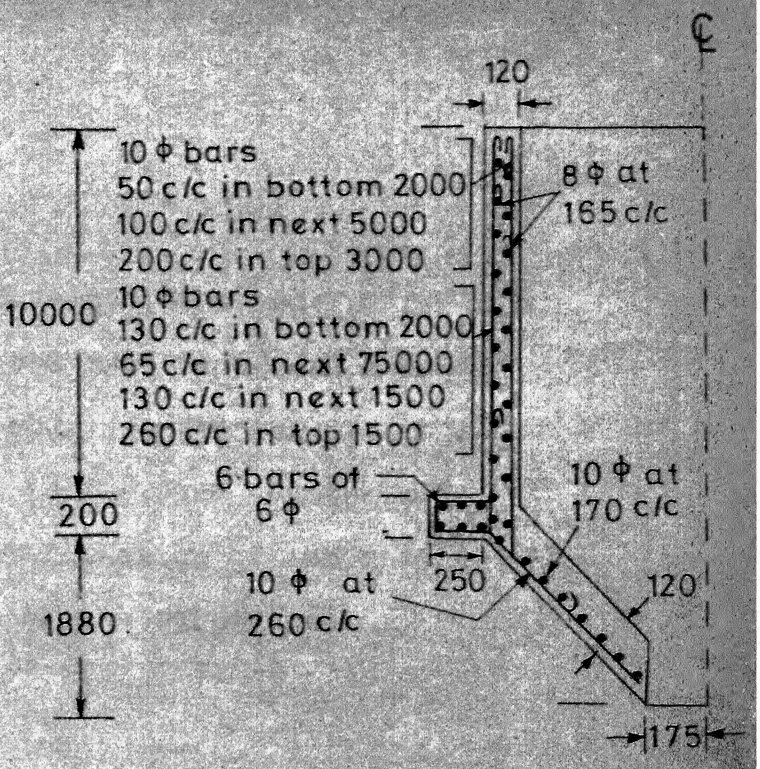


Fig.6·3·6 Reinforcement details

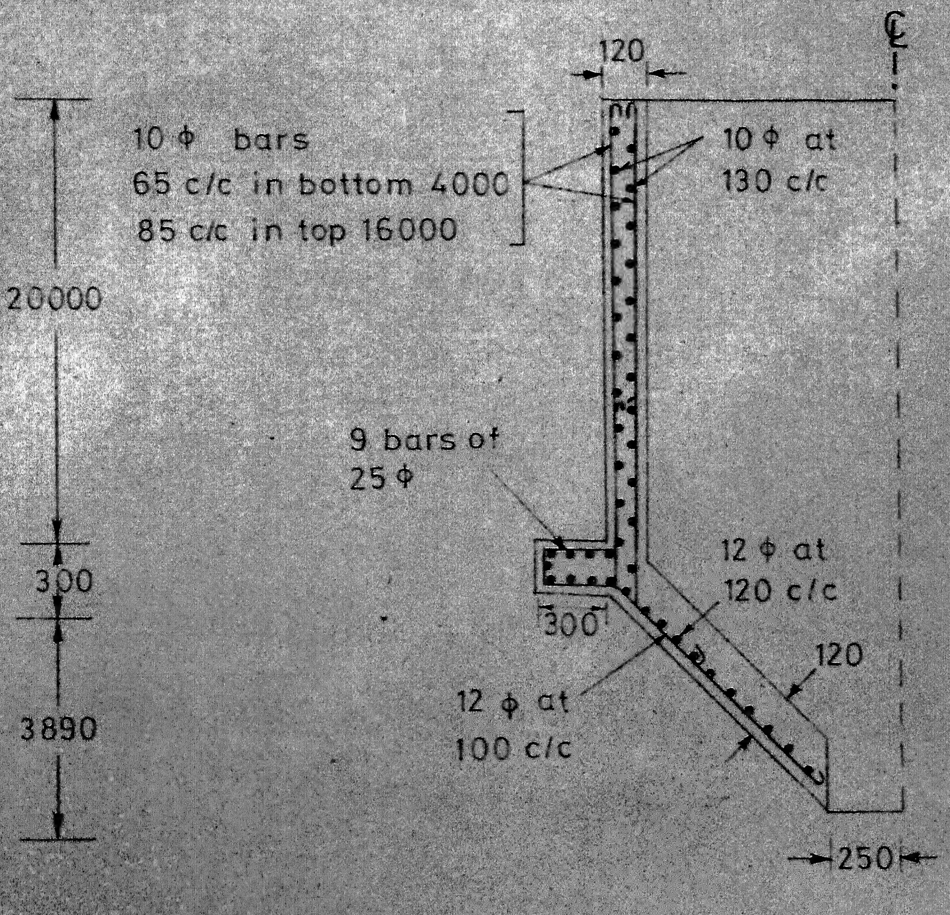


Fig.6·3·7 Reinforcement details

6.4 DISCUSSION

As the size of silo increases, the mode of failure changes from first mode to second mode or second mode to third mode. Also the cost of storing material per unit volume decreases with increase in the size of the silo. This is because of the better utilization of the material strength over a larger portion of the structure.

7. DISCUSSION OF RESULTS AND CONCLUSION

7.1 GENERAL

The basic differences between a silo and a tank are due to the nature of normal pressure distribution and the wall friction forces. In the case of a tank, storing fluid, the lateral pressure increases linearly with depth, whereas it is nonlinear in the case of a silo. In the case of the former the rate of increase in pressure with depth is constant whereas in the latter the rate of increase in pressure increases with depth. In the case of a fluid storage tank axial forces on the wall will be present only when the fluid is highly viscous, whereas in the case of silo axial forces due to friction are present for all materials like cement, coal, grain etc. In the case of fluid storage tank the difference between loading (filling) and unloading (exptying) pressure is negligible while in the case of silos the pressure on the wall during emptying process may be as high as four times that of the fillingin pressure. This pressure increase during emptving will depend not only upon material but also the conditions, namely, compact or loose packing. The silo walls assumed to be free at the top and clamped at the bottom. However,

in practice, this is not true.

7.2 DISCUSSION OF RESULTS

In all the storage structures considered herein, there are three basic modes of failure. A typical circumferential moment capacity distribution for a cylindrical wall will consist of three zones. Zone I is upto a depth e_1L wherein the nature of hoop force is compression in Zones II and III there is hoop tension. In Zones I and II the hoop force is uniform while in Zone III it is assumed to be varying linearly. The region boundary between the Zones II and III is βL from the top of the cylindrical shell. The typical modes of failure are shown in Fig. 3.3.1, wherein the dots indicate the location of the plastic hinge circle. The number of hinge circles in failure modes indicate the mode of failure. In modes one and three the hoop compression zone is absent. In the case of the third mode, the top most portion of the wall beyond the failure zone upto a depth of e_4L, any stress distribution that is statically admissible may be assumed (Fig. 3.7.1).

From equations (4.2.6), (4.3.2) , (5.1.1) and (5.2.2) one can readily see that Janssen's theory is giving a lower pressure than the Reimbert's theory for

lower value of nondimensional friction parameter (T). As the value of T increases the difference decreases and for $T > 2.6$ the Janssen's theory will give more pressure than Reimbert's theory. This difference increases for T around 6.5 and after this , the difference starts decreasing and for T more than 20 this difference is almost negligible. The Reimbert's theory is found to give better correlation between theoretical and experimental data. However in this thesis only Janssen's theory, that is widely adopted, has been considered for working out a design example for $T = 2.52$ at which the difference in pressure between these two theories is almost negligible. So for smaller value of T the Reimbert's theory is conservative when compared with the Janssen's theory but for large value of T this is not the case.

From Fig. 3.8.1 it is clear that e_2 and e_5 are always greater than β while e_1 and e_4 are always less than β. This is true for $\beta \leq 0.5$ and α upto 0.75. Hence, these curves are smooth and changes in slopes are noticed only at critical points. It is also clear that the hoop compression is confined only in the top 10 percent depth of the tank wall.

From Figs. 4.7.2 and 4.7.3 it is clear that e_1, e_2, e_4 and e_5 are always less than β for $\beta \leq 0.5$ and α upto 0.75.

Hence, these curves are smooth and changes in slopes are noticed only at critical points. In Fig. 4.7.2 the curves for $\alpha = \beta = T = 1$ and $\alpha = 0.75$, $\beta = 0.5$ and $T = 1$ intersect each other at the nondimensional shell parameter CC = 280. This is due to the different failure modes. For the first curve the failure mode is the third one while for the latter it is in second mode.

In Figs. 5.6.2 and 5.6.3 it has been found that for the case of $\alpha = 0.75$ and $\beta = 0.5$ for $T = 1$ and 2.5 for second mode of failure there is a sudden drop in nondimensional pressure parameter (p_o^*). Prior to the drop the value of e_2 is less than that of β and it is greater than β after the drop. As the value of nondimensional friction parameter (T) increases, the range of nondimensional shell parameter (CC) for second mode of failure decreases. Besides, when it approaches the third mode there is a sudden rise in nondimensional pressure parameter (p_o^*). For $T = 1$ at CC=76 there is a sudden drop in p_o^* and for a value of CC = 288 there is not only a jump in p_o^* but also a change in the mode of failure. The corresponding values for $T = 2.5$ are CC = 48 and 93 respectively.

In both the cases of silo it has been noticed that the values of e_o, e_1, e_2, e_4 and e_5 decrease as the value

of T increases. This decrease in e_2 for the case of $\alpha = 0.5$ and $\beta = 0.25$ is faster than the decrease in the rate of other cases.

From section 6.3 it is clear that for a given L/R ratio the cost of storing per unit volume of material decreases as the silo becomes deeper for $\alpha = \beta = 1$. For L/D = 3.33, the unit cost for the three types of silos (shallow, medium and deep) are Rs. 278.28, Rs. 142.27 and Rs. 116.12 respectively.

On comparing the results shown in Tables 4.1 to 4.3 and 5.1 to 5.3 it is obvious that the critical values of CC (junction modes) for Reimbert's pressure distribution theory are greater than that of Janssen's. As the value of T decreases the difference between the CCR values for the two theories will decrease. This can be inferred the forementioned tables.

7.3 CONCLUSION

For a very deep silo both the Reimbert's and Janssen's theories will yield almost the same results. For either of the pressure distribution, for a given nondimensional shell parameter (CC) and nondimensional friction parameter (T), one can readily determine the mode of collapse by using the Figs. 4.7.1 to 4.7.3 and

5.6.1 to 5.6.3. For a given size of tank one can readily determine the mode of collapse from Fig. 3.8.1.

The same set of figures mentioned above for predicting the failure mode can be used to design a storage structure in conjunction with the yield condition described by equations(2.4.1) to (2.4.4), as outlined in Section 6.3.

The failure modes in the case of water tank and the silo are similar. For the first time,plastic analysis of silos is presented with variable circumferential reinforcement along the depth. Other conditions permiting, the deeper the silo the better it is in terms of economy (cost per unit volume of material stored).

A shell of given dimension may fail in different modes for different pressure distribution theories. Hence, one should be extremely careful in selecting a particular theory. Such a theory should mathematically represent the physical situation.

7.4 SCOPE FOR FURTHER STUDY

Silos covered at the top may be analysed.

Limit state design of the silo is the next logical step, wherein the collapse and the servicebility

requirements are considered.

The degree of fixity at the junction of the cylindrical silo wall and the conical hopper should be studied in depth.

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